Vector stream function method for 3D hydrodynamic stability problems

Jānis Priede

Applied Mathematics Research Centre,
Coventry University, Coventry, CV1 5FB, UK
e-mail: j.priede@coventry.ac.uk

ABSTRACT

Numerical modelling and, especially, the stability analysis of 3D hydrodynamic problems is substantially complicated by the solenoidity constraint to be satisfied by the flow of incompressible liquid. Physically, the incompressibility condition is satisfied through the pressure, which, thus, plays a role of kinematic constraint rather than being an independent dynamic variable. As a result, there are no physical boundary conditions for the pressure, which gives rise to certain computational problems. A number of schemes have been developed to circumvent these problems in direct numerical simulations using the pressure and velocity as the natural variables. In the linear stability analysis, it is preferable to eliminate the pressure in order to reduce the number of variables and, thus, the size of the associated eigenvalue problem. This can be done in several ways. Firstly, one can employ Galerkin-type approximation with solenoidal base functions. Such an approach, however, is not only an algebraically complicated, but also limited to simple and explicit boundary conditions. Secondly, the solenoidity constraint can be satisfied explicitly by the so-called velocity potentials. The latter approach is also mathematically complicated and limited to simple geometries.

This study considers an alternative approach using a vector stream function for the representation of solenoidal velocity fields in 3D hydrodynamic stability problems. Similarly to the velocity, only two components of the stream function are mutually independent. Since the stream function, as the magnetic vector potential in the electrodynamics, is determined up to a gradient of arbitrary function, a Coulomb-type gauge can be imposed on the stream function by requiring it to be solenoidal. On the one hand, this gauge decouples the components of the stream function in its relation with the vorticity, which is defined by a Poisson equation. On the other hand, the two independent components of the stream function remain coupled through the boundary conditions for the velocity. These coupled boundary conditions, which preclude construction of Galerkin-type basis, can be satisfied using a collocation approximation in combination with the capacitance matrix techniques. This reduces 3D hydrodynamic stability problem to a standard complex matrix eigenvalue problem for two independent stream function components. The proposed method is considerably simpler and much easier to implement than the Galerkin or velocity potential method.

The implementation of the method is demonstrated using the Chebyshev collocation approximation for 3D linear stability analysis of both rectangular duct and cylindrical Couette-Taylor flows. The convergence and accuracy of the method are shown to be comparable to those of the the Galerkin method. However, in contrast to the Galerkin method, the present approach may produce spurious unstable modes as the Reynolds number exceeds few thousands. The ways to reduce and, eventually, eliminate these spurious modes are discussed.