Vortices in a cylindrical domain with a localized heating

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ABSTRACT

Thermal convection has been shown to be determinant in the formation and intensity of some meteorological events such as dust devils and cyclones: dust devils are likely to form in the presence of large horizontal temperature gradients [1], and the evolution of hurricane intensity depends, among other factors, on the heat exchange with the upper layer of the ocean under the core of the hurricane [2-3].

We consider a fluid in a cylinder heated from below with a Gaussian profile, the governing equations are the incompressible Boussinesq Navier-Stokes equations coupled with a heat equation. For the numerical implementation, non-linearities are treated with Newton's method. For the discretization (for basic state and linear stability analysis) we use a spectral method by expanding the fields in Chebyshev polynomials and evaluating at the Gauss-Lobatto points [4-5].

In a previous work [6] we proved that vertical vortices (stable states with a non zero azimuthal velocity component), very similar to a dust devil, can be generated from a convective instability in a cylindrical annulus non-homogeneoulsly heated from below. In this work we show that under certain thermal and geometrical conditions, a stable vortex appears as a solution of the system also in the cylindrical domain. We analyze the stability properties of these vortical structures and we prove that the horizontal temperature gradient at the bottom of the cylinder and the vertical temperature gradient, determine the intensity of the vortex formed and its behavior can be controlled thermally by cooling or heating adequately the bottom boundary. These results connect with that observed for the evolution of the intensity of cyclones and dust devils.

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