# Structural instability of a barotropic flow leading to the emergence of a zonal jet

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## ABSTRACT

Large scale, zonal jets, that are maintained by the very eddy field they support, are commonly observed in planetary, turbulent flows [1]. Regarding the generation of mean flows in turbulent fluids, there are two main approaches: turbulent cascades and wave-mean flow interaction mechanisms. The first approach is based on the work of Rhines [2] and proposes that zonal jets emerge due to an inverse anisotropic energy cascade, a result of eddy-eddy interactions that are local in wavenumber space. The second approach initially described in the seminal paper of Eliassen and Palm [3], is based on wave-mean flow interaction mechanisms that are non-local in wavenumber space. According to the Eliassen-Palm theorem, the eddies flux momentum into or out of the mean flow in regions of stirring, or in regions where they are dissipated or absorbed at critical levels. Divergence of their momentum fluxes can maintain or suppress the mean flow. However, a question remains on whether such a mechanism can support a mean flow in cases where stirring is homogeneous, rather than localized as in the case of homogeneously forced turbulence.

In this work we address this question using the tools of Stochastic Structural Stability Theory (SSST;[4]). In SSST, the method to obtain the distribution of the eddy fluxes associated with a given jet, is provided by a linear stochastic turbulence model. The average momentum flux divergence, forces the mean flow to produce a closed non-linear system governing the joint evolution of the mean flow and the associated eddy statistics. Fixed points of this non-linear system represent steady mean flows in equilibrium with the mean eddy forcing and dissipation. Instability of these equilibria, brings about structural reconfiguration of the mean flow and the eddy statistics [4].

Using SSST, the structural instability of a barotropic fluid with no mean velocity, subjected to a homogeneous stochastic excitation is examined. Focus is on the role of the eddy mean flow feedbacks in the instability giving rise to a zonal jet. We find that the eddy-mean flow dynamics can be split into two distinct processes: eddy vorticity advection and eddy propagation due to the mean vorticity gradient. Eddy vorticity advection by the local shear, in the manner described by the Orr dynamics, produces upgradient vorticity fluxes. On the other hand, eddy propagation in the manner described by Rossby wave dynamics under the local mean vorticity gradient, produces downgradient vorticity fluxes. For a mean flow with large width compared to the eddy scale, shearing of the eddies acts exactly as anti-diffusion, whereas eddy propagation acts exactly as hyper-diffusion. If the eddies have scales smaller that a certain cut-off scale and are excited with an amplitude larger than a certain threshold, the upgradient fluxes dominate leading to structural instability and the emergence of a mean zonal jet through a bifurcation of the non-linear system.

## REFERENCES

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