The optimal path to turbulence in shear flows

Antonios Monokrousos*, Alessandro Bottaro†, Luca Brandt*,
Andrea Di Vita† and Dan S. Henningson*
* Linné Flow Centre, KTH Mechanics, SE-100 44 Stockholm, Sweden
e-mail: luca@mech.kth.se
†DICAT, Università di Genova, Via Montallegro 1, 16145 Genova, Italy

ABSTRACT

Transition to turbulence in shear flows is still one of the most challenging problems in hydrodynamics. In this paper we determine the initial condition on the laminar/turbulent boundary closest to the laminar state using nonlinear optimisation for plane Couette flow. Resorting to the general evolution criterion of non-equilibrium systems we optimise the route to the statistically steady turbulent state, i.e. the state characterised by the largest entropy production. This is the first time information from the fully turbulent state is included in the optimisation procedure. We demonstrate that the optimal initial condition is localised in space for realistic flow domains, larger than those previously used. We investigate the transition path and show how localised perturbations evolve into bent streaks that later break down to turbulence.

We employ Lagrangian optimisation where the functional $\mathcal{L}$ to maximise consist of an objective function and two constraints (the Navier–Stokes equations and the energy level of the initial disturbance),

$$\mathcal{L} = \mathcal{J} - \int_0^T (\mathbf{u}^*, NS(\mathbf{u}))dt - \lambda(\|\mathbf{u}(0) - \mathbf{U}\|^2 - \epsilon_0), \tag{1}$$

where $\mathbf{u}^*$ and λ are the Lagrange multipliers, NS the nonlinear Navier–Stokes equations, and ϵ_0 the kinetic energy of the perturbation at $t = 0$; $\mathbf{u}$ is the velocity vector and $\mathbf{U}$ the Couette base flow. The objective function is the time-averaged viscous dissipation.

In the figure bellow the energy threshold necessary to reach a turbulent state is displayed for each initial condition found by the optimisation procedure. Each of these initial conditions is labelled with the energy level ϵ_0 used in the definition of the Lagrangian. The threshold level is determined by a classic bisection procedure with an accuracy of five digits. For the largest ϵ_0 considered, one can reduce the amplitude of the initial condition and still reach the turbulent state. When decreasing the constraint on the initial energy, we reach a value, ϵ_{0T}, below which the flow remains laminar for any $t < T$. This is indeed the nonlinear optimal initial condition of smallest amplitude leading to a statistically steady turbulent flow.