Validation and Verification of a 3D Instability Analysis Methodology

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ABSTRACT

The present work is motivated by the desire to study the stability of three-dimensional flows in complex geometries. A very limited number of studies of this problem exists presently, two of which are in the prototype incompressible cubic lid-driven cavity (LDC) flow [1, 2]. Both works employed low-order finite-difference and finite-volume schemes, respectively, and $O(10^6 - 10^7)$ discretization points for convergence. In the present work, a high-order finite difference scheme [3] is used in conjunction with matrix formation, storage and inversion to achieve converged results on $O(10^5)$ grid points.

Verification and validation work of our algorithm is performed on a spanwise periodic LDC flow, for which the BiGlobal instability analysis result is discussed, e.g. in [4]. TriGlobal analysis assumes a three-dimensional steady laminar basic flow and superposed unsteady perturbations of the form

$$\mathbf{Q}(\mathbf{x},t) = \bar{\mathbf{q}}(\mathbf{x}) + \epsilon\tilde{\mathbf{q}}(\mathbf{x},t); \quad \tilde{\mathbf{q}}(\mathbf{x},t) = \hat{\mathbf{q}}_{3d}(x,y,z)\exp(\sigma t), \quad \epsilon \ll 1. \tag{1}$$

The objective of the present work is to recover, via solution of the *three-dimensional* eigenvalue problem resulting from Ansatz (1), the results delivered by the BiGlobal theory assumption

$$\tilde{\mathbf{q}}(\mathbf{x},t) = \hat{\mathbf{q}}_{2d}(x,y)exp[i(\beta z - \sigma t)], \tag{2}$$

which explicitly imposes periodicity along z and solves a *two-dimensional* eigenvalue problem.

Results of the three-dimensional eigenvalue problem at $Re = 10^3$ were obtained on a desktop computer with $8\,Gb$ RAM, employing $Nx = Ny = 46$ and a tenth-order finite-difference scheme in the x and y directions, alongside $Nz = 16$ Fourier collocation points in the spanwise direction, the latter used to discretize a spanwise length $L_z = 2\pi/15$. Isosurfaces of the three-dimensional amplitude functions $\hat{\mathbf{q}}_{3d} = (\hat{u},\hat{v},\hat{w},\hat{p})^T$ of the first (stationary) and second (traveling) eigenmodes are shown in figures 1 and 3, respectively. For comparison, the corresponding two-dimensional amplitude functions $\hat{\mathbf{q}}_{2d}$, obtained by solution of the BiGlobal eigenvalue problem at $(Re,\beta) = (10^3, 15)$, using 100^2 discretization points are shown in figures 2 and 4, respectively.

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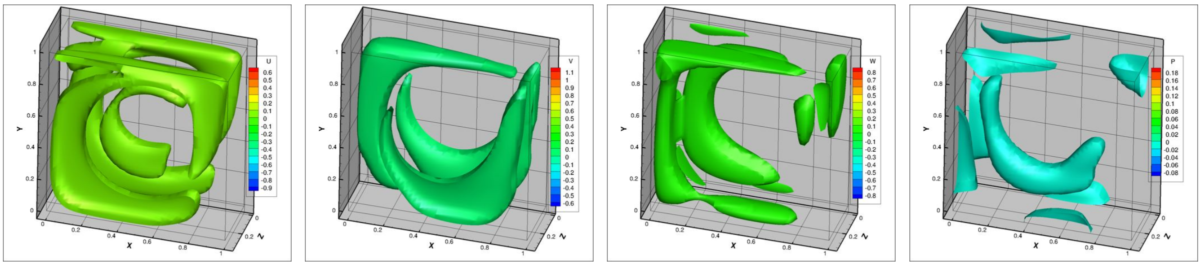

Figure 1: Amplitude functions corresponding to the most unstable stationary TriGlobal eigenmode (S1). Left-to-right: $\hat{u}(x, y, z), \hat{v}(x, y, z), \hat{w}(x, y, z), \hat{p}(x, y, z)$.

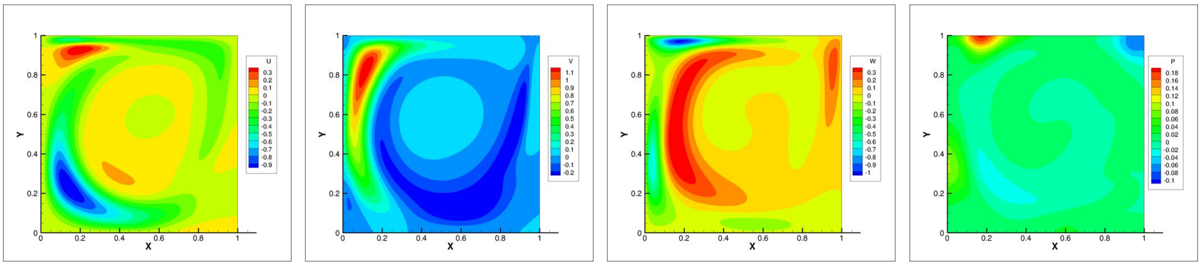

Figure 2: Amplitude functions corresponding to the most unstable stationary BiGlobal eigenmode (S1). Left-to-right: $\hat{u}(x, y), \hat{v}(x, y), \hat{w}(x, y), \hat{p}(x, y)$.

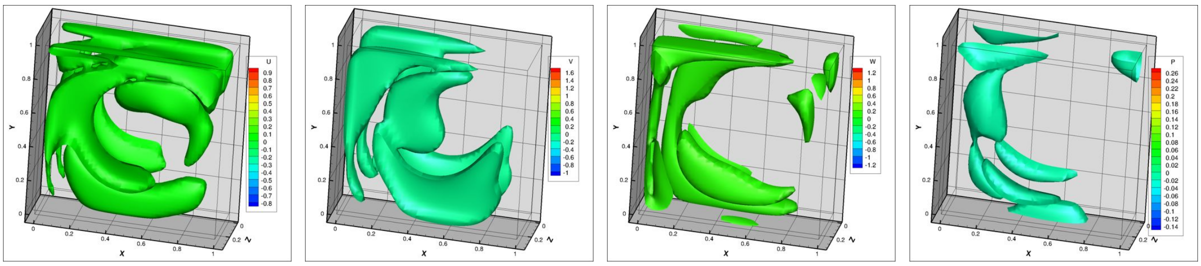

Figure 3: Amplitude functions corresponding to the most unstable traveling TriGlobal eigenmode (T1). Left-to-right: $\hat{u}(x, y, z), \hat{v}(x, y, z), \hat{w}(x, y, z), \hat{p}(x, y, z)$.

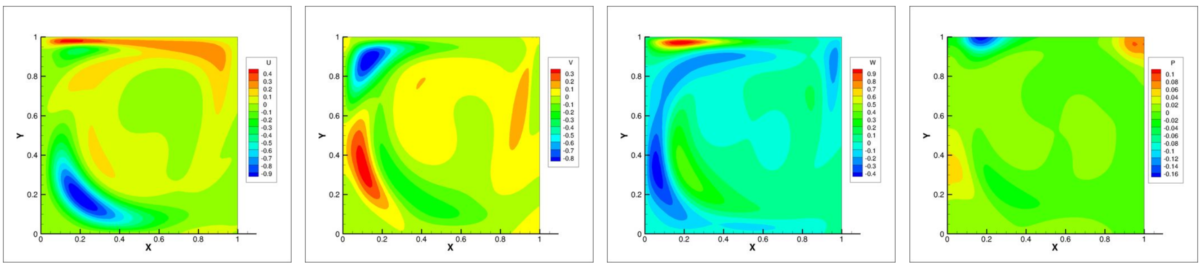

Figure 4: Amplitude functions corresponding to the most unstable traveling BiGlobal eigenmode (T1). Left-to-right: $\hat{u}(x, y), \hat{v}(x, y), \hat{w}(x, y), \hat{p}(x, y)$.