The stability of the liquid lining in fluid-conveying curved tubes

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ABSTRACT

The pulmonary airways are lined with a thin liquid film. Surface tension at the air-liquid interface aims to re-distribute the fluid in the liquid lining such that the interfacial area is minimised. In straight cylindrical vessels and in the absence of any air flow, the air-liquid interface can adopt a cylindrical shape (a surface of constant mean curvature), corresponding to a film of uniform thickness. Pulmonary airways are slightly curved, however, and Jensen [1] showed that in such tubes surface tension re-distributes the lining fluid such that it accumulates on the outer wall of the curved vessel while dry spots develop on the inner wall.

In this study we demonstrate that the surface-tension-driven migration of fluid towards the outer wall of a curved vessel can be opposed by the azimuthal shear stresses generated by the Dean-like secondary flows that develop when air is driven along the vessel. Assuming the pressure-driven flow along the curved vessel to be fully developed, we employ a combination of numerical and asymptotic approaches to demonstrate that the competition between the two effects allows the existence of steady solutions in which the liquid lining has finite thickness along the entire perimeter of the vessel. We study the stability of these steady solutions and show that they cease to exist (via a saddle-node bifurcation) when the surface tension is reduced below a certain critical value. For smaller surface tensions, fluid is always found to be driven towards the inner wall and dry spots develop on the outer wall, via a mechanism first studied in reference [2].

REFERENCES

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