Constitutive equations for granular flows under uniform mean shear and external torque

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ABSTRACT

Collective motions of granular materials behave like fluid motions under appropriate conditions. However, unlike the Newtonian fluids, the basic equations for the collective motions of granular materials have not been well established yet. One possible way to treat such granular flows is to model them as flows of a micropolar fluid, a fluid with polar micro-structures such as spin.

Here, we focus on the constitutive equations of granular flows in which the spin field is not subordinate to the vorticity field. Numerical simulations of two-dimensional granular flows under uniform shear and external body torque were performed. Uniform mean shear field and mean spin field, which is not subordinate to the vorticity field, are realized in the simulations.

The estimates of stresses based on kinetic theory by Lun[1] are in good agreement with the simulation results for a low area fraction $\nu = 0.1$ but the agreement becomes weaker as the area fraction gets higher. However, the estimates in the kinetic theory can be fitted to the simulation results up to $\nu = 0.7$ by renormalizing the coefficient of roughness. For a relatively dense granular flow ($\nu = 0.8$) near the jamming transition point, the simulation results are also compared with Kanatani's theory[2]. It is found that the dissipation function and its decomposition into the constitutive equations in Kanatani's theory are not consistent with the simulation results.

The present study suggests that there is a regime of relatively dense granular flows as micropolar fluid, where physical pictures based on neither kinetic theories nor Kanatani's theory is adequate. The regime is located near the jamming transition point. In order to understand the mechanism of the transition, it would be necessary to draw a new microscopic description of particle interaction that is different from mutually independent short-time collisions in the kinetic theory or long-time contacts with sustained velocity difference in Kanatani's theory.

REFERENCES

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