A robust hybrid direct/iterative solver for pseudo-arclength continuation of 3D Navier-Stokes problems

Jonas Thies*, Fred Wubs†, Henk Dijkstra‡

* Centre for Interdisciplinary Mathematics
Uppsala University, P.O. Box 480, 75106 Uppsala, Sweden
e-mail: jonas@math.uu.se - Web page: http://www2.math.uu.se/cim/

† Johann Bernoulli Institute for Mathematics and Computing Science
University of Groningen, P.O.Box 407, 9700 AK Groningen, The Netherlands
e-mail: f.w.wubs@rug.nl - Web page: http://www.math.rug.nl/cmnm

‡ Institute for Marine and Atmospheric Research Utrecht
Utrecht University, Princetonplein 5, 3584 CC Utrecht, The Netherlands
e-mail: dijkstra@phys.uu.nl - Web page: http://www.phys.uu.nl/∼dijkstra

ABSTRACT

Pseudo-arclength continuation with inexact Newton linearization is a powerful technique for investigating parameter sensitivity of complex dynamical systems. For three-dimensional fluid dynamics problems the application of this technique is still challenging because of the ill-conditioned linear systems with the Jacobian that have to be solved. The indefiniteness of the matrix and the computational complexity makes direct solvers inefficient in 3D. Iterative solvers typically lack robustness at higher Reynolds numbers (when eigenvalues start crossing the imaginary axis).

We present a hybrid direct/iterative solver for the Jacobian of the incompressible Navier-Stokes equations on structured grids. The method is based on domain decomposition, where the variables in the interior of the subdomains are eliminated using a direct method and the Schur-complement on the separators is solved using a robust iterative method. The method features a grid-independent convergence rate and does not break down at high Reynolds numbers. The structure-preserving preconditioning technique allows recursive application of the algorithm, although we only present a two-level variant here.

The method is applied for the first time to a problem in cylindrical coordinates (the Taylor-Couette problem) to demonstrate its practical applicability and as a first step towards geophysical fluid dynamics applications such as ocean flows. Robustness and parallel performance are demonstrated by directly computing highly unstable steady states of this application and the lid-driven cavity problem in two and three space dimensions.

REFERENCES

[1] F.W. Wubs, J. Thies, *A robust parallel ILU solver with grid-independent convergence for the coupled steady incompressible Navier-Stokes equations*. In J.C.F. Pereira and A. Sequeira (Eds). Proceedings 5th ECCOMAS CFD 2010, 2010

[2] F.W. Wubs, J. Thies, *A robust two-level incomplete factorization for (Navier-)Stokes saddle point matrices*. Available as preprint arXiv:1006.1874, 2010