An efficient continuation algorithm to compute steady state bifurcation diagrams in large size coupled fluid flow and heat transfer problems

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M. Medale*, **P. Cerisier**†
IUSTI, UMR 6595 CNRS-Aix-Marseille University, 13453 Marseille, France.
* e-mail: Marc.Medale@polytech.univ-mrs.fr, † e-mail: Pierre.Cerisier@polytech.univ-mrs.fr

ABSTRACT

Path-following algorithms also designated continuation methods aim at computing branches of steady state solutions for a given range of control parameters. Several softwares have been developed to deal with small size continuation and bifurcation problems involving Ordinary Differential Equations (see e.g. Doedle [1]). Unfortunately, most of interesting coupled fluid flow and heat transfer problems are governed by Partial Differential Equations and result in large size algebraic systems that can no longer be addressed by standard methods or packages. One way to overcome this difficulty is to implement algorithms and methods that could achieve reasonable scalability on parallel supercomputers.

This communication presents one of such algorithms based on an Asymptotic Numerical Method that enable to compute steady state bifurcation diagrams of large size problems for coupled Navier-Stokes and heat transfer solutions in any polygonal or circular based containers. It includes a continuation algorithm that accurately locate the primary bifurcation point (threshold of convection) along with any subsequent secondary bifurcations. Furthermore, this continuation algorithm is implemented on a high performance computer environment to deal with non linear thermo-convective instability problems [2].

The application problem concerns surface-tension-driven-convection in containers of small-to-moderate aspect ratios ($\Gamma \leq 30$) and various shapes, filled with a high Prandtl number silicon oil ($Pr = 900$) and topped by a thin air layer ($Bi = 0.1$). This parametric study is intended to analyse the aspect ratio influence on: (i) the threshold of convection and its pattern, (ii) the nature of that bifurcation, (iii) the location of subsequent secondary bifurcations and their associated patterns. The computations have been first validated versus reference solutions in parallelepiped or circular container [3, 4].

One of the most amazing results of this study is the influence of the container size and shape on the nature of the bifurcation at threshold. As an illustration, let us consider the square container case, it has been found to be either transcritical or saddle node depending on the aspect ratio. Moreover, the computations reveals the existence of many steady-state bifurcations in the vicinity of the threshold for small-to-moderate aspect ratios. The patterns inherits the container symmetries at threshold, but progressively looses them as going through subsequent secondary bifurcations, which occur closer to the threshold for higher aspect ratios.

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