Unsteady flow in a curved tube

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ABSTRACT

The unsteady flow in a curved vessel is a common model for the blood flow in the arteries and air flow in the bronchi and bronchioles. For developing flow in a curved pipe, the meridional flow, induced by the interplay of viscous drag, centrifugal forcing and pressure gradients, leads to a “collisional” boundary-layer flow, which can provoke a singularity in the boundary-layer equations at a finite distance along the pipe. Understanding the role and consequences of these types of singularity in the full Navier–Stokes equations is the motivation behind the present work.

We initially consider a related, but non-physiological, problem of the temporal evolution of viscous, incompressible fluid in a rotating torus of finite curvature, after an impulsive change in rotation frequency. In this case, analytic progress is facilitated by the simple form of the initial flow. We show that (rotationally symmetric) eruptive singularities of the boundary layer can occur at the inner and outer bend of the pipe for a decrease or increase in rotation rate, respectively. Comparison with finite (but large) Reynolds number computations shows good agreement with the asymptotic predictions and the eruption is characterised by ejection of fluid from the boundary layer in to the core. We also show that the flow is subject to a non-axisymmetric inflectional instability induced by the developing eruptive instability and in qualitative agreement with the experimental observations of Madden & Mullin (1994) [1].

The more complex structure of the core flow in the pressure-driven problem hampers analytic progress, but numerical results will be presented.

REFERENCES

[1] Madden, F. N. & Mullin, T. *The spin-up from rest of a fluid-filled torus.* J. Fluid Mech., **265**, 217 (1994).