Oscillatory flow of Maxwell and Oldroyd-B fluids: theoretical analysis and experiments

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ABSTRACT

In this work we revisit the generic problem of wall-bounded oscillatory flow of a viscoelastic liquid. We focus on the periodic steady state of laminar flow, at small Re. In this approximation the governing equations are linear and therefore accessible to analytical treatment. We use as constitutive equations the upper convected Maxwell model (UCM) and the more general Oldroyd-B model, that includes a Newtonian solvent contribution. We consider two basic geometries (rectangular and cylindrical), under two different modes of driving. In the rectangular geometry the fluid motion is induced by the synchronous oscillation of two parallel infinite walls, while in the cylindrical one the flow is induced by the oscillatory motion of a bottom piston and a top endwall. In both cases transverse shear waves are generated in the fluid. We show that the flow is characterized by the damping length of the amplitude of oscillation of the shear waves, x_0, and their wavelength, $\lambda_0/(2\pi)$. For a Newtonian fluid these two lengths are equal and coincide with the thickness of the boundary layer, leading to overdamped shear waves. However, for viscoelastic fluids $x_0 > \lambda_0/(2\pi)$ and underdamped shear waves can propagate effectively before they are attenuated [1]. In 'narrow' systems (small setup dimensions compared to x_0) the viscoelastic shear waves extend through the whole system and superpose themselves originating an interference pattern inside the fluid domain. This leads to a resonant behaviour with a huge increase of the velocity amplitude at particular frequencies. However, as we add an increasing Newtonian solvent contribution to the model, measured by the viscosity ratio $X = \eta_s/\eta$, the resonances progressively disappear and the magnitude of the velocity and shear rate are drastically reduced [2].

We compare our theoretical predictions to experimental results that we have obtained for oscillatory flow in a cylindrical tube with large aspect ratio. The experimental velocity profiles are measured for a wormlike micellar solution, CPyCl-NaSal [100:60], using a time resolved PIV technique. At low amplitudes and frequencies of the driving oscillation the profiles follow the predicted trends. However, at high velocity amplitudes (high shear rates) the experimental profiles deviate from the theoretical predictions, as a result of the nonlinearities of the fluid rheology in this regime. At even higher drivings (Wi > 1) the parallel shear flow eventually becomes unstable leading to the formation of symmetric vortices inside the tube [3]. The observed instability presents a mild hysteresis that reveals the subcritical nature of this first bifurcation.

REFERENCES

[1] Mitran, S.M., Forest, M.G., Yao, L., Lindley, B. & Hill, D.B. 2008., *Extensions of the Ferry shear wave model for active linear and nonlinear microrheology.*, J. Non-Newtonian Fluid Mech. **154**, 120–135.

[2] Andrienko, Y.A., Siginer, D.A. & Yanovsky, Y.G. 2000. *Resonance behavior of viscoelastic fluids in Poiseuille flow and application to flow enhancement.*, *Int. J. Non–Linear Mech.* **35**, 95–102.

[3] Torralba, M., Castrejón-Pita, A.A., Hernández, G., Huelsz, G., del Río, J. A., & Ortín, J. 2007. *Instabilities in the oscillatory flow of a complex fluid.*, *Phys. Rev. E* **75**, 056307:1–9.