Motion of an Expanding, Spherical Gas Bubble in a Viscous Liquid under Gravity when the Centre Moves in a Vertical Plane

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ABSTRACT

The basic differential equation for the growth of the radius R(t) with time t of a spherical gas bubble, expanding adiabatically in an incompressible and inviscid fluid, was obtained by Rayleigh [1]. Plesset [2] modified Rayleigh's equation taking into account the viscosity of the fluid surrounding the gas bubble. Rayleigh's formula has been modified when the translation of the bubble through the surrounding inviscid liquid is considered and the deviation of the bubble shape from its spherical form is also taken into account [3]. This last study has been further modified [4] when the liquid through which the bubble moves is taken to be viscous but the bubble shape is restricted to be spherical in form.

In all the above studies the presence of gravitational acceleration has been ignored, and the bubble centre has been considered to move along a straight line in references [3], [4] and [5]. Chakraborty and Tuteja[6] investigated the influence of gravitational acceleration on the results in bubble dynamics obtained in References [1], [2], [4] and [5]. We consider here the motion of the bubble when the bubble centre will not now move along a vertical line, but will move in a vertical plane in the presence of gravitational acceleration, acting vertically downwards .We find the path and velocity of the bubble centre and the growth of radius of the bubble as time progresses.

The radius R(t) of the bubble is found to vary periodically with time when the acceleration due to gravity is small. But as the acceleration due gravity increases, this periodicity in the value of R(t) with t is lost. This tendency is reversed when we study the influence of viscosity on the bubble motion. It is observed that the radius R(t) of the bubble varies periodically with time when the viscosity is large, but as the viscosity decreases, this periodicity in the value of R(t) with time t is lost.

The path traced by the centre of the bubble as time progresses is obtained by solving the differential equations of the problem. The case when $\dot{R} = 0$ [non-expanding bubble] and when $\dot{R} \neq 0$, [expanding bubble] are compared. The effect of buoyancy is felt more when $\dot{R} \neq 0$, since for an expanding bubble the upward thrust felt due to buoyancy is more, as the

volume of the displaced liquid is more, the radius R of the expanding bubble being larger than that for a non-expanding bubble [$\dot{R} = 0$]. The acceleration of the bubble centre as well as its velocity in the vertical direction are, therefore, expected to be larger for an expanding bubble [$\dot{R} \neq 0$] than for a bubble of constant radius [$\dot{R} = 0$]. The path of the bubble centre in the former case is expected to rise vertically earlier than in the latter case and results of numerical computations confirm this expectation.

REFERENCES

[1] Lord Rayleigh: On the pressure developed in a liquid during the collapse of a spherical void, Phil. Mag. (Ser 6) 34, 94 (1917).

[2] Plesset M.S. and Prosperetti, A.: Bubble dynamics and cavitation, Annu. Rev. Fluid Mech. (eds) M. Van Dyke, J.V. Wehauscn and J.L. Lumley (Annual Review, Palo Alto) 9, 177 (1977).

[3] Chakraborty, B.B.: Effect of fluid flow outside a nearly spherical, expanding gas bubble on the growth of its radius, Phys. Fluids 30, 2903 (1987).

[4] Chakraborty, B.B.: Effect of a viscous fluid flow past a spherical gas bubble on the growth of its radius, Proc. Ind. Acad. Sciences 100, 185 (1990).

[5] Chakraborty., B.B. and Tuteja G.S.: Motion of an expanding, spherical gas bubble in a viscous liquid under microgravity, Microgravity Science and Technology

[11] Chakraborty., B.B. and Tuteja G.S.: Motion of an expanding gas bubble in a viscous liquid under gravity, Phys. Fluids 5, 187 (1993)