Stability and Bifurcation Diagram of Boussinesq Thermal Convection in a Rotating Spherical Shell

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ABSTRACT

Global thermal convection is considered to take place in stellar interiors, the atmospheres of the giant planets, and the fluid cores of the terrestrial planets, due to internal heat sources and/or external cooling. The problem of Boussinesq thermal convection in a rotating spherical shell is one of the most fundamental frameworks to investigate characteristics of these phenomena.

The onset of thermal convection in a rotating spherical shell has been investigated analytically and numerically for half a centuty. The critical parameters and the critical convection patterns has been revealed for a wide parameter range. Finite-amplitude thermal convection in a rotating spherical shell has also been studied through numerical time integrations in a certain range of parameters[1], but global behaviour of the solutions is not well understood.

In this study, in order to understand fundamental properties of thermal convection in a rotating spherical shell, we explore finite-amplitude thermal convection solutions by Newton method rather than numerical time integrations, and examine their stability systematically. The radius ratio of the inner and the outer spheres, and the Prandtl number are set to be 0.4 and 1, respectively, the commonly used values in the previous studies. The Taylor number, which is proportional to the rotation rate squared, is varied from 52^2 to 500^2, and the Rayleigh number is increased from the critical values R_c to at most $2R_c$.

It is revealed that the finite-amplitude propagating solutions have four-fold symmetry in the azimuthal direction (TW4) and stably exist when the Rayleigh number is between the critical value R_c and $1.2 \sim 2R_c$, which depends on the Taylor number. It is also found that, in this parameter range, all the unstable modes of TW4 are subharmonic modes.

Near the critical point, the propagating direction of TW4 solution continuously changes from retrograde to prograde as the Taylor number increases. The switching of the propagating direction is interpreted by expansion and contraction of vortices arising from their meridional structure[2].

The propagating velocity of TW4 increase continuously as the Rayleigh number increases when the Taylor number is small, however, the propagating velocity decrease continuously when the Taylor number is large. In particular, when the Taylor number is between 340^2 and 500^2, the propagating direction changes from prograde to retrograde as the Rayleigh number increases. The decrease of the propagating velocity can be interpreted as an advection by mean zonal flows induced by the nonlinear effects of the thermal convection[3].

Recently, we study the torques on the inner and outer boundaries due to TW4s in the same parameter range, and it is revealed that when the Taylor number is smaller than 200^2 the inner boundary is torqued in the prograde direction, while it is in the retrograde direction for the Taylor number larger than 300^2. The magnitude of the torque is significant for the inner sphere with the density of the same order of magnitude as the fluid.

REFERENCES

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