Shear instability in a stratified fluid when shear and stratification are not aligned

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ABSTRACT

The three-dimensional stability of a two-dimensional plane Bickley jet of width L and maximum velocity U_0 in a stably stratified fluid of constant Brunt-Väisälä frequency N is examined in an inviscid and Boussinesq framework. The plane of the jet is assumed to be inclined with an angle θ with respect to the vertical direction of stratification. The stability analysis is performed using both numerical and theoretical methods for all the values of θ and Froude number $F = U_0/(LN)$.

We first obtain that the most unstable mode is always a 2D Kelvin-Helmholtz (KH) sinuous mode. The condition of stability based on the Richardson number $Ri > 1/4$, which reads here $F < 3\sqrt{3}/2$, is recovered for $\theta = 0$. But, as soon as $\theta \neq 0$, that is when the directions of shear and stratification are not perfectly aligned, the Bickley jet is found to be unstable whatever the Froude number. We show that two modes are involved in the stability properties. We demonstrate that when F is decreased below $3\sqrt{3}/2$, there is a "jump" from one 2D sinuous mode to another. For small Froude number, we show that the characteristics (oscillation frequency, growth rate, wavenumber) of the most unstable mode vary with the inclination angle θ as $\sin\theta$.

The property of the modes to exhibit a cross-stream oscillating structure at infinity, that is, of being radiative, is also analysed. We show that there exists a family of radiative modes with growth rates that may be comparable to the growth rate of the most unstable KH mode. The characteristics of the most unstable radiative modes are obtained. The most unstable radiative modes are shown to be 2D for $F > 3\sqrt{3}/2$ but to possess a spanwise wavenumber for $F < 3\sqrt{3}/2$. The main features of these 3D radiative modes are found to be well-captured by a WKBJ asymptotic analysis in the limit of large spanwise wavenumbers.