Liquid Film Flow Over Heated Porous Surfaces

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ABSTRACT

Gravity-driven flow down an inclined plane has been studied extensively, for example by Ruyer-Quil and Manneville [1], Balmforth and Mandre [2], and D'Alessio *et al.* [3]. Film flow down an inclined plane with bottom heating has been considered by Kalliadasis *et al.* [4] using the integral-boundary-layer approach, and by Trevelyan *et al.* [5] using the weighted residual model for both a specified bottom temperature and heat flux. This problem was revisited by D'Alessio *et al.* [6] for the combined case of heating and a sinusoidal bottom profile using the weighted residual model. The stability associated with flow down a porous inclined plane was investigated by Pascal [7] using the Orr-Sommerfeld equation, and by Pascal and D'Alessio [8] for a wavy inclined surface using the weighted residual model. Recently, Sadiq *et al.* [9] studied flow over an inclined plane with both bottom heating and permeability.

The problem studied in this work simultaneously considers heating, permeability, surface tension and sinusoidal topography using the weighted residual model. Heating occurs as a result of a temperature difference between the bottom surface and the ambient air. The saturated permeable bottom is modelled using the Beavers and Joseph [10] slip condition. The model equations for film flow, expressed in terms of the flow rate, film thickness and free-surface temperature, are used to predict the critical Reynolds number for the onset of instability over an even bottom. In addition, the corresponding Benney equation is derived and is also used to determine the instability threshold. When compared, the two results were found to agree to $O(\delta_1)$, where δ_1 is a small parameter that describes the permeability of the bottom surface. The critical Reynolds number indicates that both heating and permeability destabilize the flow, and the combined effect is to further destabilize the flow. The results were also compared with those obtained by Sadiq *et al.* [9], and again, agreement to $O(\delta_1)$ was achieved. Due to approximations inherent in the model, this level of agreement is expected. For a sinusoidal bottom, Floquet theory was applied to analyze the influence of bottom topography on the stability of the flow. The key finding is that bottom topography has a stabilizing effect on the flow for weak surface tension with and without heating and permeability.

An important advantage offered by the model equations over the complete set of equations and boundary conditions is that the model equations can be solved numerically much faster than the full equations. A fractional step method was used to numerically advance the model equations in time. Nonlinear numerical simulations were carried out to verify the predictions of the linear theory and to compute the evolution of the interfacial waves for unstable flows. Lastly, numerical solutions of the full Navier-Stokes and energy equations, obtained using the CFX software package, were also conducted and contrasted with the model solutions.

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