Numerical tools for the stability analysis of 2D flows

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ABSTRACT

This work deals with the numerical stability analysis of 2D Navier-Stokes equations. The steady solutions are determined by using an Asymptotic Numerical Method (ANM)[1] which consists on the association of a perturbation method and the finite element method. By this way, the nonlinear solutions are analytically determined without a great amount of computational times. To study the stability of these solutions, some bifurcation indicators are introduced. Two kind of instabilities can be studied: steady bifurcation points[2] and Hopf bifurcation points[3].

In this study, a particular attention is given to the steady bifurcation points, which correspond for example to the loss of symmetry in channel flows with sudden expansion. In this case, the critical Reynolds numbers of the flow are determined by an indicator which have the property to be null at the bifurcation points. This indicator, which is a scalar, is computed by using a perturbation method during the determination of the nonlinear solutions of the steady Navier-Stokes equations[2]. By this way, no additional computations are required to study the stability of the solutions. Once these critical Reynolds numbers are determined, a continuation method is proposed to determine all the nonlinear bifurcating branches emanating from these singular points. As jacobian matrices are singular in these points, specific numerical methods, based also on the ANM, are introduced to easily determined the nonlinear solutions. The stability of these solutions is determined by a single eigenvalues computation. From these solution branches, one can also look for additional steady or Hopf bifurcation points. All these numerical methods are automatic and just require two user parameters which are the accuracy of the nonlinear solutions ant the truncature order of the asymptotic expansions. These numerical tools are applied to classical examples, such as flow in symmetric channels with a sudden expansion. These examples permit to show the reliability of the proposed numerical algorithms.

REFERENCES

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