Viscous fingering under elastic membranes

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Viscous fingering in Hele-Shaw cells in which viscous fluid, contained in the narrow gap between two rigid plates, is displaced by a less viscous fluid is an archetype for front-propagating, pattern-forming phenomena [1,2]: if the less viscous fluid is injected at a sufficiently fast rate so that viscous forces exceed surface tension forces, the axisymmetric interface between the two fluids is linearly unstable to non-axisymmetric perturbations. The nonlinear growth of this instability causes the development of distinct fingers whose tips become subject to a similar instability, leading to so-called tip-splitting. Repeated tip-splitting combined with the arrest of the interface after the passage of the finger tips ultimately creates a complex dendritic pattern as shown in Fig. 1a.

(a) (b) (c)

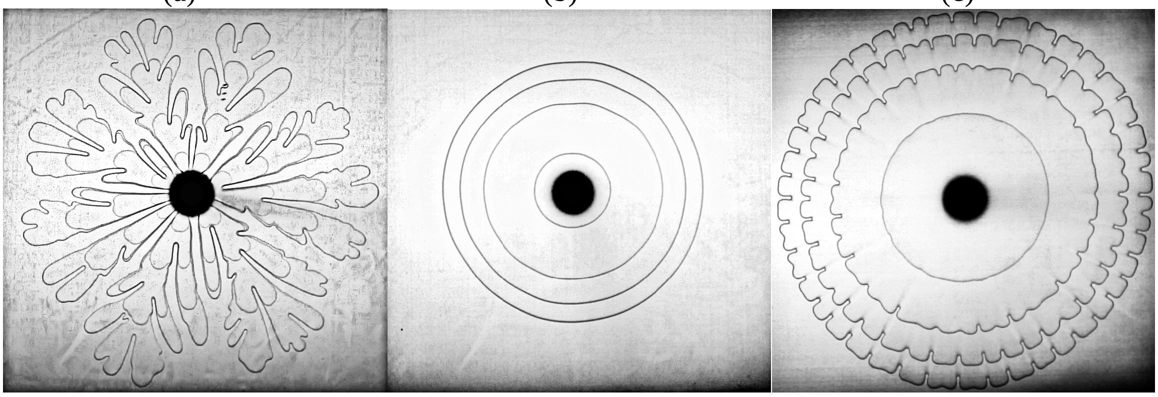

Figure 1: Top view of front propagation in a radial Hele-Shaw cell. Air injected at a constant flow rate in the centre displaces silicone oil with a viscosity of 1.0 kg $m^{-1}s^{-1}$. Four successive positions of the interface are shown in each image, with the smallest interface recorded at t=0.4 s after air injection started: (a) rigid cell, flow rate Q=145 ml/min, Delta t = 0.53 s; (b) elastic cell with Latex membrane (thickness h=0.3 mm), Q=145 ml/min, Delta t = 4.7 s; (c) elastic cell with Latex membrane (thickness h=0.3 mm), Q=1836 ml/min, Delta t = 0.8 s. The initial thickness of the fluid layer is b_0=0.55 mm.

We show that elastic deformations of the plates that bound the fluid can have a dramatic and unexpected effect on the onset and nonlinear development of this instability (see Fig. 1). In an elastic-walled Hele-Shaw cell where one of the bounding plates is replaced by a latex membrane, we find that the instability is suppressed and the interface remains axisymmetric (see Fig. 1b) for values of the injection rate at which the rigid system exhibits strongly nonlinear interfacial growth (see Fig. 1a). Moreover, the critical injection rate beyond which the axisymmetrically expanding interface becomes unstable to non-axisymmetric perturbations in the elastic-walled system (see Fig 1c) is 10^2-10^3 times larger than the corresponding predicted by Paterson [2] for the rigid system. We find that the interface in the elastic cell becomes unstable beyond a critical flow rate that decreases as $K^{-1/4}$ (K is the bending stiffness of the membrane), and we propose a physical mechanism for damping that agrees with our experimental findings.

REFERENCES

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[2] L. Paterson. Radial fingering in a Hele-Shaw cell. J. Fluid Mech., 113:513-529, 1981.