Multifractal Probability Density Function Theory and Its Application to Fully Developed Turbulence

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ABSTRACT

Multifractal Probability Density Function Theory (MPDFT) is a statistical mechanical ensemble theory for analyzing instability of systems providing fat-tail PDFs under the assumption that the singularities due to the instability distribute themselves multifractal way in real physical space. The degree of singularity for those quantities which are responsible for intermittent phenomena is specified by the singularity exponent α that appears in the scale transformation of the Navier-Stokes equation as an arbitrary parameter taking real values. The A&A model are written down with three parameters which are fixed by the condition of energy conservation, the definition of μ and the *scaling relation* that relates between the entropy index q, appeared in the definition of Rény entropy or Tsallis entropy, and $\alpha_\pm$ the zeros of the multifractal spectrum $f(\alpha)$, i.e., $f(\alpha_\pm) = 0$. The multifractal spectrum is uniquely related to the distribution function for α. The distribution function is responsible for the tail part of PDFs of those observable quantities revealing intermittent behavior.

In order to extract the intermittent character of the fully developed turbulence due to the instability, it is necessary to have information of hierarchical structure of the system. This is realized by producing a series of PDFs for responsible singular quantities with different length $\ell_n = \ell_0 \delta_n$ with $\delta_n = \delta^{-n}$ ($n = 0, 1, 2, \cdots$) and $\delta > 1$ that characterize the sizes of regions in which the physical quantities are coarse-grained. The value for δ is chosen freely by observers. Therefore, the choice of δ should not affect the theoretical estimation of the values for the fundamental quantities characterizing the turbulent system under consideration. The framework of MPDFT itself provides us with a scaling relation, $(\ln 2)/(1 - q) \ln \delta = 1/\alpha_- - 1/\alpha_+$, as a proper relations to satisfy above requirement.

We will present a conjecture that the scaling relation given above provides us with a new interpretation of turbulence, i.e., the system of fully developed turbulence consists of the accumulation of the δ-scale Cantor sets characterized by δ^∞ periodic orbits of a non-linear dynamical systems with different values of δ. The δ^∞ periodic orbits appear in a δ periodic window in which there are infinite δ^k periodic windows ($k = 2, 3, 4, \cdots$), one fitting inside another. Each δ^k periodic window ($k = 1, 2, 3, \cdots$) starts with the δ^k period saddle-node bifurcation (δ times ramification) followed by the periodic doubling bifurcation and then by the chaotic region where the δ^{k+1} periodic window nests. Observation of the system with a magnification δ extracts the information of the δ-scale Cantor sets constituting the turbulence.

REFERENCES

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