Near-wall bubble oscillations in an ultrasonic pressure field

Sergey A. Suslov*, **Richard Manasseh**†,‡ **and Andrew Ooi**‡

* Mathematics, FEIS, Swinburne University of Technology
Hawthorn, Victoria, 3122, Australia
e-mail: ssuslov@swin.edu.au, web page: http://www.swin.edu.au/mathematics/staff/Sergey-Suslov

† FEIS, Swinburne University of Technology
Hawthorn, Victoria, 3122, Australia
e-mail: rmanasseh@swin.edu.au, web page:
http://www.swin.edu.au/engineering/staff/Richard-Manasseh

‡ Department of Mechanical Engineering, University of Melbourne
Melbourne, Victoria, 3010, Australia
e-mail: a.ooi@unimelb.edu.au - web page: http://www.melbourne.edu.au/

ABSTRACT

We consider oscillations of microscopic bubbles caused by the ultrasonic pressure field that arise in various biomedical applications such as ultrasound imaging or targeted drug delivery [1,2]. When bubbles approach a blood-vessel wall, it is known to modify their dynamic response. The wall effects on small-amplitude oscillations of a big bubble have been investigated in [2]. To study the wall influence on nonlinear dynamics of microbubbles, we adopt the Keller-Miksis-Parlitz equation [3]. This base model describes the time evolution of the bubble surface, which is assumed to remain spherical. The major feature distinguishing this model from the classical Rayleigh-Plesset equation [4] is the inclusion of the effect of acoustic radiation losses owing to liquid compressibility in the momentum conservation. Two situations are considered: the base case of an isolated bubble in an unbounded medium and a bubble near a solid wall. In the latter case the wall influence is modeled by including a symmetrically oscillating image bubble. The bubble dynamics is traced using a numerical solution of the model equation, similarly to earlier studies [5]. Subsequently, Floquet theory is used to accurately detect the bifurcation point where bubble oscillations stop following the driving ultrasound frequency and undergo period-changing bifurcations. Of particular interest is the situation when a bifurcation leads to apparent period tripling. The parametric bifurcation maps are obtained as functions of the driving frequency, pressure amplitude and the distance from the wall. It is shown that the presence of the wall generally stabilizes the bubble dynamics so that its oscillations follow the driving pressure waves at larger values of the pressure amplitude.

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