# PLASTIC DAMAGE MODEL FOR NONLINEAR REINFORCED CONCRETE FRAMES ANALYSIS

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**Summary.** This paper describes an improved analytical model for predicting the nonlinear response of multistory reinforced concrete frames, based on the concepts of isotropic continuum damage mechanics combined with plasticity theory. A frame member is considered to consist of an elastic beam with two inelastic hinges at its ends, according to the conventional lumped plasticity models. This model uses the conventional plastic damage, adapted to the case of frames.

#### 1 INTRODUCTION

Continuum mechanics is still not the most suitable analysis framework of certain civil engineering structures, in spite of its evolution in recent decades, especially in damage mechanics. In many cases, the structures are modeled as trusses or frames, while the continuum mechanics is used only for relatively simple structures. However, plasticity theory has been successfully adapted to frame analysis by using the concept of lumped plasticity models, in which it is assumed that plastic effects can be concentrated at special locations called plastic hinges. In the case of frame analysis, the plastic hinges are located at the end of the beams of the structure.

Using the same concept of lumped plasticity model, Flórez<sup>1</sup> develops an adaptation of the damage models to frame analysis where the damage is limited to the extremities of the beam, in such a way that the damage is concentrated on plastic hinges.

The objective of this paper is to employ plastic-damage models in frame analysis, with application to reinforced concrete structures, in accordance with the classic theories of continuous damage mechanics and the classic theories of plasticity. What distinguishes this work from others is the fact the complete plastic-damage constitutive model is here implemented into a frame analysis algorithm.

### **2 GENERAL CONCEPTS**

Let us consider a plan frame with b elements, connected in i nodes. The generalized deformations of the beam b,  $\Phi_b$ , can be described as function of the displacement vector  $\mathbf{U}$  and of the displacement transformation matrix  $\mathbf{B}_b^{-1}$ :

$$\mathbf{\Phi}_{b} = \mathbf{B}_{b}\mathbf{U} \to \mathbf{\Phi}_{b}^{T} = \begin{bmatrix} \phi_{i} & \phi_{i} & \delta \end{bmatrix}$$
 (1)

where  $\phi_i$  and  $\phi_j$  indicate, rotations of the member at the ends i and j with respect to the chord i-j, respectively, and  $\delta$  is the elongation of the chord with respect to its length in the initial configuration (Fig. 1).

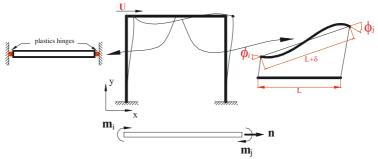


Figure 1: Representation of the deformation, internal forces and lumped dissipation model of a frame.

If we consider now the existence of plastic hinges concentrated at the end of the beam b, is necessary to define a new internal variable, the plastic deformations vector, which contains the plastics rotation at the joint i and j, respectively:

$$\left(\mathbf{\Phi}_{b}^{p}\right)^{T} = \begin{bmatrix} \phi_{i}^{p} & \phi_{j}^{p} & 0 \end{bmatrix} \tag{2}$$

The member deformation can now be expressed as:

$$\mathbf{\Phi}_{b} = \mathbf{F}_{b}^{e} : \overline{\mathbf{M}}_{b} + \mathbf{\Phi}_{b}^{p} \quad \text{or } \overline{\mathbf{M}}_{b} = \mathbf{S}_{b}^{e} : \left(\mathbf{\Phi}_{b} - \mathbf{\Phi}_{b}^{p}\right)$$
(3)

where  $\mathbf{F}_b^e$  and  $\mathbf{S}_b^e$  denote the flexibility and local elastic stiffness matrices, respectively. The generalized effective stress vector  $(\mathbf{\overline{M}}_b)^t = [m_i \ m_j \ n]$  contains the final forces inside the member, thus  $m_i$  and  $m_j$  are the moments at the ends of the member and n indicate the axial force (see Figure. 1).

In the case of damage variables, we can define the damage vector as:

$$\mathbf{D}^T = \begin{pmatrix} d_i & d_j & d_a \end{pmatrix} \tag{4}$$

where the parameters  $d_i$  and  $d_j$  measure the flexion damage of the hinges i and j, respectively. Parameter  $d_a$  is the measure of the axial damage of the member. These variables can taske values between zero (no damage) and one (completely damaged).

In the case of reinforced concrete structures, the plasticity is generally associated

physically with the yielding of steel, while damage occurs with the cracking and the posterior failure of the concrete. Only in very advanced stages, when concrete shows the highest level of cracking, the damage can start in the steel. This kind behavior allows studying separately the evolution of the variables related with damage and plasticity. Using the same idea proposed by Simo and Ju<sup>2</sup>, this model uses two independent multipliers and two potentials for damage and plasticity, each one with its own law of evolution. The coupling occurs during the determination of the final generalized stress vector, which is defined as:

$$\mathbf{M}_{b} = (\mathbf{1} - \mathbf{D}_{b}) : \overline{\mathbf{M}}_{b} \tag{5}$$

Recalling the definition of  $\overline{\mathbf{M}}_b$  (equation (3)) and considering that  $(1-\mathbf{D}_b): \mathbf{S}_b$  can be defined as  $[\mathbf{S}^d(\mathbf{D}_b)]$ , which is the local stiffness matrix of a damaged member, equation (5) can be rewritten as:

$$\mathbf{M}_{b} = \left\lceil \mathbf{S}^{d} \left( \mathbf{D}_{b} \right) \right\rceil : \left( \mathbf{\Phi}_{b} - \mathbf{\Phi}_{b}^{p} \right) \tag{6}$$

It can be seen that for  $\mathbf{D}_b$  equal to zero, that is no damage at all, we obtain the standard stiffness matrix of an elastic member. If one of the flexural damage parameters takes a value equal to one and the other damage parameter takes a value equal to zero then  $\left[\mathbf{S}^d\left(\mathbf{D}_b\right)\right]$  becomes the stiffness matrix of the elastic member with an internal hinge at one end, on the left or on the right side. When both flexural damage parameters are equal to one, then we have the stiffness matrix of an elastic truss beam, only with axial force.

### 3 APPLICATION

With the intention of analyzing the validity of the proposed model, we calculated the reinforced concrete frame shown in Figure 2, which was tested by Vecchio and Emara<sup>3</sup>. The testing process consisted in applying a total axial load of 700 kN to each column and maintaining this load in a force controlled mode throughout the test. A lateral load was then applied, in a displacement controlled mode, until the ultimate capacity of the frame was achieved.

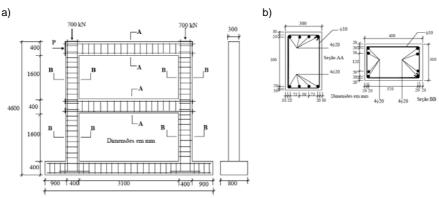


Figure 2: Details of the test frame<sup>3</sup> - a) the frame dimensions and details, b) Sections A and B

Beams	Columns
E=2.633E+7kN-m	E=2.633E+7kN-m
$A=0.12 m^2$ , $I=1.6E-3 m^4$	$A=0.12 \text{ m}^2, I=1.6E-3 \text{ m}^4$
Mp=161  kNxm, Mu=189  kNxm	Mp=253  kNxm, Mu=273  kNxm

Table 1: Parameters used in the model.

The parameters used in the model are given in table 1. In this case, the plastic constitutive equation used only bending moments<sup>4</sup> while the lineal damage equation<sup>5</sup> has been considered for to determine the damages variables evolution, using in this case the fracture energy equal as g = 100 for all members.

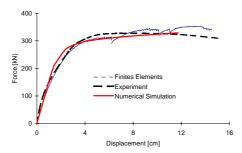


Figure 3: Comparison between numerical results and testing.

In the Figure 3 we can see the comparison between the numerical results using the proposed model and the experimental results. The results obtains using a finite elements program<sup>6</sup> are also included in this figure.

#### 12 CONCLUSIONS

- This proposed model is an effective tool for the numerical simulation of the collapse of frames.
- It is are alternative for the case when other types of analyses, such as those based on multi-layer models, appear to be computationally expensive or impractical due to the size and complexity of the structure.

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