A NON-ROTATING ANISOTROPIC DAMAGE MODEL FOR BRITTLE MATERIALS

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Summary. *A theoretical model is presented suitable for the description of the damaging process in brittle materials. Damage is defined by a second-order symmetric tensor and activates at different orientations to the principal strain directions, according to the sign of the principal strain which attains a damage threshold. The capabilities of the model in describing the mechanical response of material elements subjected to non-proportional stresses are illustrated.*

1 INTRODUCTION

The failure structures made of brittle materials is accompanied by local loss in stiffness and strength, as a consequence of the concurrent coalescence and growth of microcracks: a phenomenon which is usually defined as ‘damage’. A damage model was previously developed by the authors[1,2] to simulate the nonlinear mechanical behaviour of brittle materials, and successfully applied to structural analyses of ancient masonry towers. The model was originally conceived to describe the time evolution of damage in brittle materials which can be macroscopically assumed to be isotropic in the undamaged state, such as rubble-like masonry and concrete, under either increasing or sustained 3D stresses. Damage is characterized by a second-order tensor, whose principal directions (which are somehow associated with the normal to any plane microcrack) do not rotate throughout the stress history. In the original model, the first microcrack at any point in the solid activates perpendicularly to one of the principal strain directions, as such strain attains a given threshold. In this work, a distinction is introduced regarding the activation of cracks in tension and compression (Sec. 2). A maximum of three orthogonal microcracks can activate at any point, so that the damage-induced anisotropy is, in the most general case, orthotropy. Sec. 3 illustrates the essential features of the model in the simulation of simple tests on a material element. Finally, in Sec. 4 possible future developments of the model are outlined.

2 THE DAMAGE MODEL

Let $\boldsymbol{D}$ be a second-order symmetric tensor, which is supposed to characterize the damage state of any material element in an initially isotropic solid. The principal directions of the damage tensor are denoted by x_α ($\alpha = I, II, III$). The principal components of $\boldsymbol{D}$, D_α, are supposed to be given by

$$D_\alpha = 1 - \frac{1}{1 + A_H \left\langle y_{\alpha\alpha} - y_{0H} \right\rangle^{B_H}}, \tag{1}$$

where A_H, B_H, y_{0H} are material parameters; the latter one is a damage threshold. These parameters take different value in compression ($H = C$) and tension ($H = T$), according to the sign of the direct strain along x_α. This law is similar to that proposed in[3] to reproduce the uniaxial stress-strain curves in tension and compression for concrete. In this model the damage-driving variable is an equivalent strain measure ($y_{\alpha\alpha}$), which is one of the direct components of a non-dimensional 'damage force' tensor, $\boldsymbol{y} = ½\, \boldsymbol{\varepsilon}\, \boldsymbol{\varepsilon}$. Denoting by $\boldsymbol{n}_\alpha$ the versor of any axis x_α, $y_{\alpha\alpha} = \boldsymbol{n}_\alpha \cdot (\boldsymbol{y}\boldsymbol{n}_\alpha)$. Damage is irreversible ($\dot{D}_\alpha \geq 0$) and increases only if $\dot{y}_{\alpha\alpha} > 0$. When the maximum eigenvalue of $\boldsymbol{y}$ attains either one of the damage thresholds at any point of the solid, the first damage direction (x_I) is activated. If the principal strain that first attains the damage threshold is positive (tension), x_I coincides with one of the principal strain directions; if it is negative (compression), x_I activates at 45° to this principal strain in the plane defined by the extreme principal strains. A possible second damage direction may be later activated in the plane orthogonal to x_I if the greatest direct component of the damage force tensor $y_{\alpha\alpha}$, with $\boldsymbol{n}_\alpha \perp x_I$, exceeds one of the damage threshold values. The second and, eventually, the third activated damage directions are collinear with the extreme direct components of $\boldsymbol{y}$, irrespective of the sign of the direct strain along $\boldsymbol{n}_\alpha$, to preserve the orthotropic nature of the constitutive law.

Referring to any Cartesian frame (x,y,z), neglecting inelastic strains, the stress-strain law for the material can be expressed in matrix notation as $\{\varepsilon\} = [C]\{\sigma\}$, where the flexibility matrix $[C]$ is affected by damage. In the local frame of the principal directions of damage, the matrix representation of the flexibility tensor is

$$[\hat{C}] = \frac{1}{E}\begin{bmatrix} \psi^{-1}_{I,I} & -\nu\psi^{-1}_{I,II} & -\nu\psi^{-1}_{I,III} & 0 & 0 & 0 \\ & -\nu\psi^{-1}_{II,II} & -\nu\psi^{-1}_{II,III} & 0 & 0 & 0 \\ & & \psi^{-1}_{III,III} & 0 & 0 & 0 \\ & & & 2(1+\nu)\psi^{-1}_{I,II} & 0 & 0 \\ & \mathit{symm.} & & & 2(1+\nu)\psi^{-1}_{II,III} & 0 \\ & & & & & 2(1+\nu)\psi^{-1}_{I,III} \end{bmatrix}, \tag{2}$$

where E, ν = elastic constants of the virgin material, $\psi_{\alpha,\beta} = [(1-D_\alpha)(1-D_\beta)]^{½}$ ($\alpha,\beta = I, II, III$).

The principal damage direction(s) activated at any point in the solid are fixed throughout the subsequent stress history, so that the proposed model can be qualified as a 'non-rotating smeared crack model'. This peculiarity makes the model suitable for structural analyses involving non-proportional loads.

3 NUMERICAL RESULTS

The capabilities of the proposed model in reproducing the mechanical response of material elements subjected to different stress histories in the nonlinear field are now illustrated. In Fig. 1 the stress-strain plot in uniaxial tension and compression is shown, for different values of one of the damage parameters (A_C). The unsymmetric behaviour in tension and compression typical of brittle materials is correctly described. A decrease in the value of the damage parameter is matched by a decrease in compressive strength.

Fig. 2 shows the results of the simulation of two different biaxial compression stress histories, tending to a common final state. During history no. 1, first the stress along x_3 is increased (up to 4.5 MPa), then also the stress along the orthogonal axis x_1 is monotonically increased (up to failure of the material element). During history no. 2, the two stresses are applied on the material element in reverse order. The axial stress σ_3 is plotted versus the axial strain ε_3 in Fig. 2. The maximum value reached by σ_3 in the second stress history is lower than in the first one, meaning that the biaxial strength of the material is affected by the stress ratio, although the bounds on the damage forces are uncoupled in the damage criterion. In this example, the same two damage directions are activated at the end of the stress path (at ±45° to the axes x_1, x_3 in the plane $x_2 = 0$).

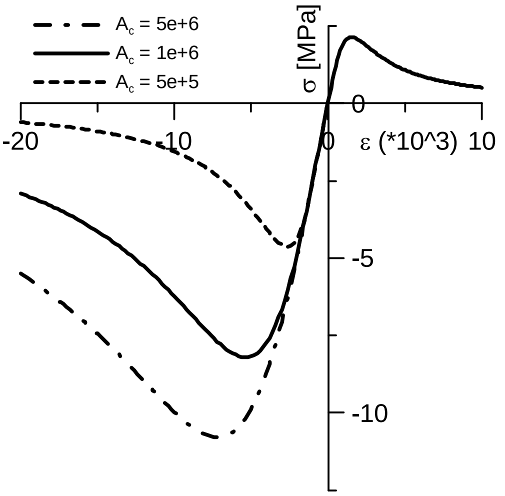


Figure 1 – Stress–strain curves in uniaxial tension/compression, for different values of the damage parameter A_C ($B_C = 1.2$, $y_{0C} = 5e\text{-}7$, $A_T = 1e{+}6$, $B_T = 1.08$, $y_{0T} = 5e\text{-}8$).

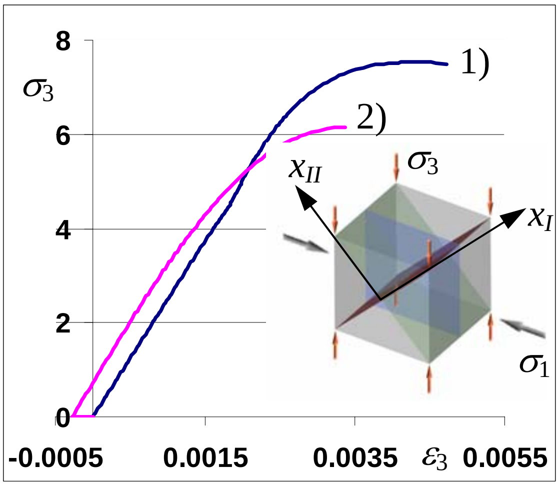
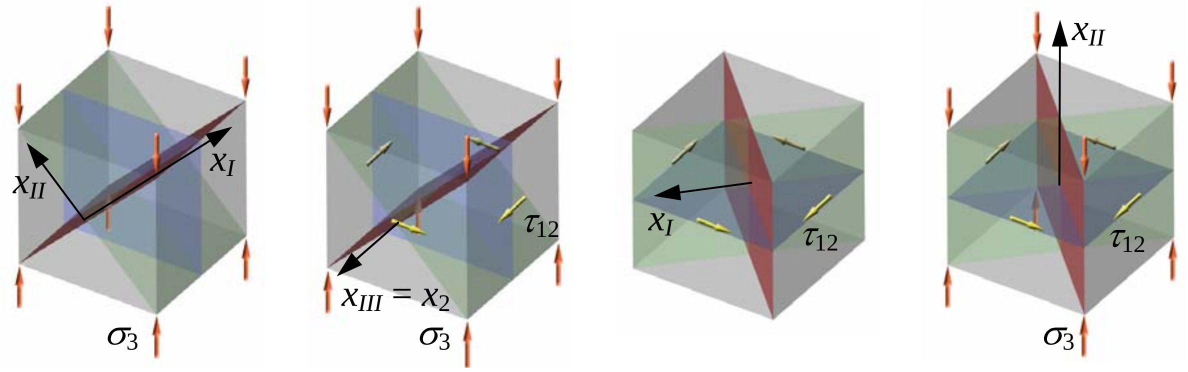


Figure 2 – Stress–strain curves in biaxial compression, according to different stress paths: 1) step 1: σ_3, step 2: $\sigma_1 + \sigma_3$; 2) step 1: σ_1, step 2: $\sigma_1 + \sigma_3$.

Finally, a numerical simulation was performed to show that, in general, the order of application of the stresses affects the ‘crack’ pattern in the material element. Two stress histories are considered. During history no. 1, the material element is first compressed along x_3 (up to a strain value $\varepsilon_3 = -0.02$), then a shear stress in the plane (x_1,x_2) is monotonically increased (up to failure of the material element). Compressing the element during the first phase activates the same damage directions as in the previous example (Fig. 3a); the subsequent shear stress activates a third damage direction orthogonal to x_2 (Fig. 3b). During history no. 2, first the shear stress τ_{12} is increased (up to 6 MPa), then the element is compressed along x_3 up to failure: during the first loading phase, only one damage direction bisecting the axes x_1 and x_2 is activated (Fig. 3c), whereas during the second phase a damage direction is activated collinear with x_3 (Fig. 3d).



(a) (b) (c) (d)

Figure 3 – Damage directions activated during two different non-proportional stress histories. History no. 1: (a) step 1: σ_3, (b) step 2: τ_{12}. History no. 2: (c) step 1: τ_{12}; (d) step 2: σ_3.

4 CONCLUDING REMARKS

The proposed damage model is capable of capturing several aspects of the nonlinear behaviour of brittle solids, namely, the damage-induced anisotropy, the unsymmetric behaviour in tension and compression, the dependence of the crack pattern on the stress history. In principle, it is possible to extend the model to allow for creep-induced damage, similarly to[1]: this would allow the safety of ancient massive buildings subjected to heavy persistent loads to be investigated through the proposed model.

Extension are still required to incorporate in the model important phenomena, such as the 'unilateral' behaviour of the material (i.e., stiffness recovery upon crack closure) and the development of irreversible (plastic) strain, which were neglected in the present version.

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