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A NEW NUMERICAL SCHEME FOR LARGE DEFORMATION NON-ISOTROPIC PLASTICITY

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Summary. *In the paper an original integration scheme for non isotropic finite deformation plasticity is reported and compared with the analogous model for isotropic materials. It is formulated in a general tensorial form, and is thermodynamically consistent. Numerical applications show that the new scheme can be effectively applied to structural problems.*

1 INTRODUCTION

Starting from the systematic approach developed by Simo [1], many works have been developed on the numerical analysis of finite deformation elastoplasticity. Multiplicative decomposition of the deformation gradient is usually adopted. Most of the works are restricted to isotropic material models, so that the so called exponential algorithm can be used. Extensions have been presented for non isochoric deformation processes [2].

In the paper a numerical model for elastoplastic materials is reported, developed without restrictions regarding the isotropic behavior of the material and the isochority of the anelastic process. It starts from the formulation of a full set of thermodynamically consistent constitutive relations and it is compared with the classical exponential integration scheme. The scheme has been implemented in a FE code, after a proper derivation of the consistent tangent operator.

1 KINEMATICS

Classical multiplicative decomposition is used, introducing an intermediate stress free configuration (anelastic configuration), splitting the deformation gradient tensor in its elastic and anelastic parts, while the velocity gradient $\mathbf{l}$ gets additively decomposed as follows

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_a \tag{1}$$

$$\mathbf{l} = \mathbf{l}_e + \mathbf{l}_a \tag{2}$$

The indices *e* and *a* stands for “elastic” and “anelastic” respectively. In order to fully describe

the deformation process, others deformation rate measures can be introduced. In particular the following convective deformation rate measures are defined in the intermediate configuration

$$\hat{\mathbf{l}} = \mathbf{F}_e^T \mathbf{l} \mathbf{F}_e = \hat{\mathbf{l}}_e + \hat{\mathbf{l}}_a \tag{3}$$

$$\hat{\mathbf{d}} = sym\left(\hat{\mathbf{l}}\right) = sym\left(\hat{\mathbf{l}}_e\right) + sym\left(\hat{\mathbf{l}}_a\right) = \hat{\mathbf{d}}_e + \hat{\mathbf{d}}_a \tag{4}$$

See [3] for details.

2 NUMERICAL MODEL

Assuming that, for a general process, the balance equations hold and that an elastic free energy potential ψ exists that rules the elastic part of the process, it is well known that the Clausius-Duhem inequality can be particularized for a general isothermal process as follows

$$D = \tau \cdot \mathbf{d} - \rho_0 \dot{\psi} \geq 0 \tag{5}$$

being D the mechanical dissipation, τ the Kirchhoff stress tensor, $\mathbf{d} = sym(\mathbf{l})$ the velocity of deformation tensor ρ_0 the (reference configuration) density and ψ the elastic Helmoltz free energy. We assume that the elastic potential depends only on the elastic part of the deformation process, $\psi = \psi(\mathbf{F}_e)$. It is well known (see [4]) that for a general elastically isotropic material the free energy must depend on the elastic deformations only through the elastic left Cauchy-Green deformation tensor defined as $\mathbf{b}_e = \mathbf{F}_e \mathbf{F}_e^T$, while if we suppose only that the elastic free energy is objective it must depend on the elastic deformations through the right Cauchy-Green deformation tensor $\mathbf{C}_e = \mathbf{F}_e^T \mathbf{F}_e$, i.e.

isotropy
$$\psi = \psi(\mathbf{b}_e) \rightarrow \dot{\psi} = \nabla_{\mathbf{b}_e}(\psi) \cdot \dot{\mathbf{b}}_e \tag{6}$$

objectivity
$$\psi = \psi(\mathbf{C}_e) \rightarrow \dot{\psi} = 2\nabla_{\mathbf{C}_e}(\psi) \cdot \dot{\mathbf{C}}_e / 2 \tag{7}$$

If we suppose that the elastic domain of the material is bounded by a regular yield function χ defined in the Kirchhoff stress space Eqs (6) e (7) and the maximum dissipation principle lead to the following sets of constitutive relations for isotropic and non isotropic materials

$$\text{isotropic mat.} \rightarrow \begin{cases} \tau = 2\rho_0 \nabla_{\mathbf{b}_e}(\psi) \mathbf{b}_e \\ L_v \mathbf{b}_e = -2\dot{\lambda} \nabla_\tau \chi \mathbf{b}_e \\ \dot{\lambda} \geq 0;\ \chi \leq 0;\ \dot{\lambda}\chi = 0 \end{cases} \qquad \text{anisotropic mat.} \rightarrow \begin{cases} \mathbf{S}_e = 2\rho_0 \nabla_{\mathbf{C}_e}(\psi) \\ \hat{\mathbf{l}}_a = \dot{\lambda} \nabla_{\mathbf{S}_e} \chi \\ \dot{\lambda} \geq 0;\ \chi \leq 0;\ \dot{\lambda}\chi = 0 \end{cases} \tag{8}$$

In Eqs (8) the symbol $L_v(\bullet)$ stands for the Lie derivative (in the velocity of deformation vector field) and $\mathbf{S}_e = \mathbf{F}_e^{-1} \tau \mathbf{F}_e^{-T}$ is the second elastic Piola-Kirchhoff stress tensor defined in the intermediate configuration. For a detailed analysis of the constitutive model see [3]. The constitutive relations for anelastic isotropic materials are generally numerically integrated in a principal stress space using the so called exponential algorithm (see. [5]), that is based on the coaxiality of the stress and the elastic and anelastic deformation tensors, so that it is usable only if the material is both elastically and anelastically isotropic, that is the elastic free energy

potential and the yield function are isotropic function of their arguments. An alternative numerical integration algorithm must be used for anisotropic materials, and it is proposed an "eulerian" algorithm. Starting form the flow rule (8b), and using definition (3) it is obtained

$$\dot{\mathbf{F}}_a = \dot{\lambda}\,\mathbf{C}_e^{-1}\nabla_{\mathbf{S}_e}\hat{\chi}(\mathbf{S}_e)\mathbf{F}_a \tag{9}$$

Assuming $\dot{\mathbf{F}}_a$ constant in the step (at odd with the classical exponential algorithm), the update equation for the anelastic deformation is found as

$$\mathbf{F}_a = \left(\mathbf{I} - \lambda\,\mathbf{C}_e^{-1}\nabla_{\mathbf{S}_e}\hat{\chi}\right)^{-1}\mathbf{F}_{an} \tag{10}$$

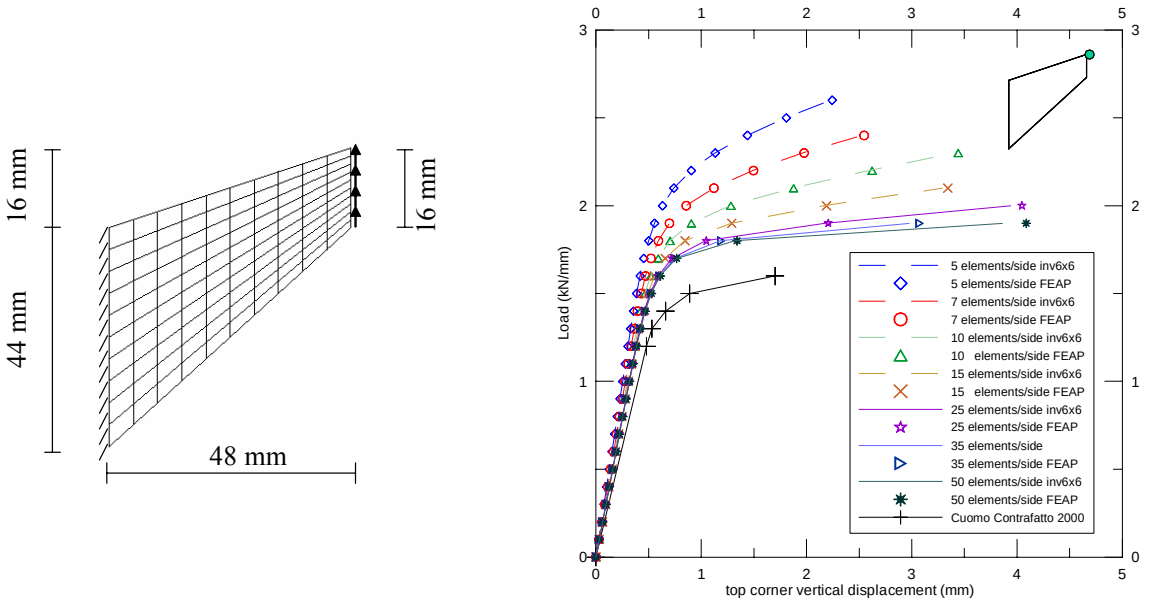
The algorithm has been implemented in a FE code, after linearization, enforcing the stress state admissibility condition $\chi(\tau)=0$ and the full tensorial compatibility condition

$$\mathbf{e} \overset{def}{=} \frac{1}{2}\left(\mathbf{I} - \left(\mathbf{f}\mathbf{f}^T\right)^{-1}\right) \rightarrow \mathbf{e} = \Delta t\left(\mathbf{d}_e + \mathbf{d}_a\right) \tag{11}$$

being $\mathbf{f}$ the step gradient of deformation tensor defining the deformation increment from the initial step configuration B_{tn} to the final step configuration B_{tn+1}
A Newton-Raphson integration scheme is employed, that allows to recover the algorithmic tangent. For details see [6]:

3 APPLICATION

The application reported in this paper concerns a test of validity of the scheme described in the previous section. Since it must hold, as a particular case, also for an isotropic material, it has been applied to the solution of a membrane made with a von Mises material.



(a) (b)

Figure 1: (a) Tapered beam geometry and (b) results comparison

The results have been compared with those obtained with a standard finite deformation plasticity code, valid only for isotropic material [FEAP]. The tapered beam is clamped on one side and loaded by a vertical uniform distributed load on the opposite one. An hyper-elastic potential depending on the third invariant of the right Cauchy-Green deformation tensor has been used [1]. The sample has been meshed with different levels of accuracy. The process grows until a transversal section of the sample is fully plasticized, when equilibrium can be no longer satisfied. The results of the analysis in terms of the vertical displacement of the top-right corner of the model vs. the load are reported, for different levels of mesh discretisation, in Figure 1(b). It is apparent that the results compare well with those obtained using standard analysis (valid, however, only for isotropic models), and the computational effort (number of iterations) is almost identical. For comparison in Figure 1(b) are also reported the results obtained by an analogous model in small deformations.

4 CONCLUSIONS

- In the paper a full set of constitutive model for elasto-plastic non isotropic materials and the numerical integration scheme are described. The numerical model has been compared with the analogous one for isotropic elastoplastic materials.
- The application of the proposed model to an isotropic case has shown that the efficiency of the proposed model is comparable to that of the exponential algorithm although it is more onerous in the implementation.

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