VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D. R. J. Owen (Eds)

ROTATION-FREE TRIANGULAR AND QUADRILATERAL SHELL ELEMENTS FOR SHEET-METAL FORMING

M. Brunet*, F. Sabourin*

* Laboratoire de Mécanique des Contacts et des Solides (LaMCoS)
Institut National des Sciences Appliquées de Lyon
20 Avenue Einstein, 69621 Villeurbanne, France
e-mail: michel.brunet@insa-lyon.fr, web page: http://lamcos.insa-lyon.fr

Key words: Shell Element, Rotation-Free, Sheet-Forming, Spring-Back, Crash-Simulation.

1 INTRODUCTION

Bending elements, plates and shells, using only out-of-plane translational degrees of freedom are not new but, so far as we know for shell analysis, with triangular shape only, the state of the art has been given by Oñate and Zarate in [1]. In the context of sheet-metal forming analysis, Brunet and Sabourin [2] [3] have proposed an approach to compute the constant curvature field within each triangle in terms of the six out-of-plane displacements of the neighbouring nodes. More recently, Sabourin and Fayet [4] have shown the kinematical aspects of the patch formed by the four triangular elements using the reciprocal screws formulations. In this paper, the formulation is extended to the 4-node quadrilateral element and based on the area quadrilateral coordinates interpolation of the rigid body angles in term of the out-of-plane nodal deflections of the patch surrounding the element and related to the bending angles along the four sides of the quadrilateral. Since warping modes are possible modes of deformation, two additional bending angles must be considered along the diagonals of the quadrilateral. The one-point quadrature is achieved at the center for the membrane strains with a warping correction. The membrane hourglass control is obtained by the perturbation or the physical stabilization procedure. Spring-back analysis of a U-shape strip sheet and the crash simulation of a beam-box complete the demonstration of the bending capabilities of the proposed rotation-free triangular and quadrilateral elements.

2 ROTATION-FREE TRIANGLE “S3”

Let us consider the triangular element (1,2,3) of Fig. 1 and its immediate neighbours and, for example, triangle (3,2,5) sharing the boundary 2-3 with the element (1,2,3). After the rotational angle α_1 for triangle (1,2,3) and α_5 for triangle (3,2,5), the two lines perpendicular to their respective mid-plane give the point C. The sketching of CH defines the bending angles θ_1 and θ_5 whose sum is assumed to be equal to $\alpha_1 + \alpha_5$ such that:

$$\alpha_1 + \alpha_5 = \theta_1 + \theta_5 \qquad \varepsilon_{n1} = -z\frac{2\theta_1}{h_1} = -z\frac{2\theta_5}{h_5} \qquad \{\varepsilon_n\} = z[H]\{\alpha\} \tag{1}$$

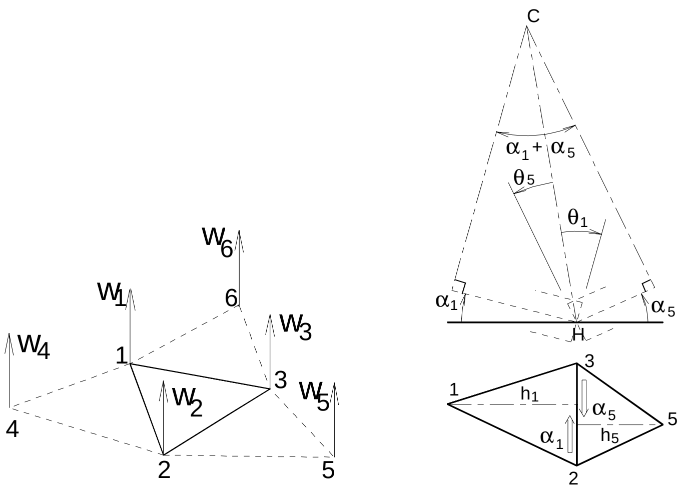

Figure 1: Triangular patch, bending and rigid body angles on common side 2-3.

Assuming that $\{\alpha\} = [C]\{w\}$ relates the rigid body rotations to the normal displacements w_i, this purely kinematical relation can be obtained by several ways. The interpolation by the triangular area coordinates A_i/A are used here since the extension to the 4-node quadrilateral with quadrilateral area functions will be straightforward:

$$w = \frac{A_1}{A} w_1 + \frac{A_2}{A} w_2 + \frac{A_3}{A} w_3 \qquad w_i = \mathrm{Cos}_z^X U_i + \mathrm{Cos}_z^Y V_i + \mathrm{Cos}_z^Z W_i \tag{2}$$

Then the rigid body rotations: $\alpha_i = \vec{grad}(w).(-\vec{n}_i)$ (3)

which leads to the bending strains in the local frame:

$$\{\delta\varepsilon\} = z[R][H][C]\{\delta w\} = z[B^b]\{\delta w\} \tag{4}$$

where [R] is the transformation matrix between the director cosines of the outward pointing normals n_i and the local x,y in plane coordinates.

3 ROTATION-FREE QUADRANGLE "S4"

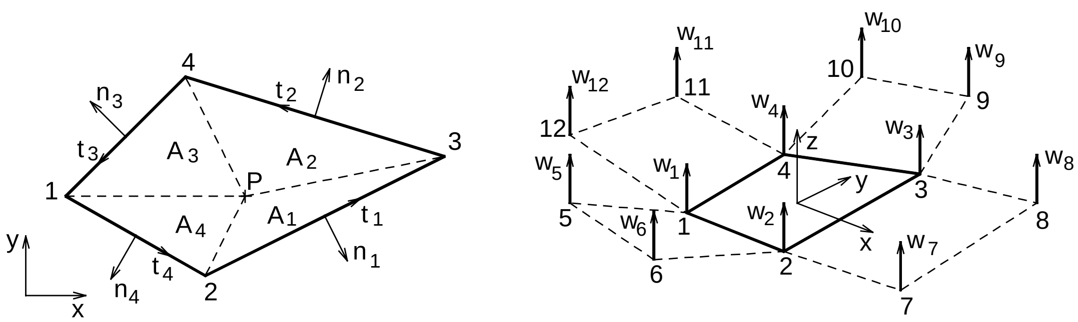

Figure 2: Projected in-plane areas and quadrilateral patch..

If it is noted that $\varepsilon_n^1, \varepsilon_n^2, \varepsilon_n^3, \varepsilon_n^4$ are the normal strains at any depth z at the mid-point of the sides joining (2-3),(3-4),(4-1) and (1-2) respectively, it is assumed for example that:

$$\varepsilon_n^1 = -2z \frac{\alpha_{2,3,4,1} + \alpha_{3,2,7,8}}{h_{4,1} + h_{1,1} + h_{7,1} + h_{8,1}} \tag{5}$$

where $\alpha_{i,j,k,m}$ means the rotational angle α at mid-side (i-j) for the quadrilateral (i,j,k,m). $h_{i,j}$ is the corresponding height from node i to side (j) where (1)=(2-3), (2)=(3-4), (3)=(4-1) and (4)=(1-2). Since quadrilateral may undergoes warping kinematic modes of deformation, additional bending strains must be taken account in the quadrilateral to avoid singular stiffness matrix. A possible way to achieve this requirement is to consider the normal bending strains along the two diagonals named (5)=(2-4) and (6)=(3-1) such that for example:

$$\varepsilon_n^5 = z \frac{\alpha_{2,4,1} + \alpha_{4,2,3}}{h_1 + h_3} \quad \text{and} \quad \underbrace{\{\varepsilon_n\}}_{6x1} = z \underbrace{[H]}_{6x12} \underbrace{\{\alpha\}}_{12x1} \tag{6}$$

where $\alpha_{i,j,k}$ is the rotational angle α along diagonal (i-j) for the internal triangle (i,j,k), h_k being the height from node k to diagonal (i-j).

Applying as for the triangular element: $\alpha_i = \vec{grad}(w).(-\vec{n}_i)$ provides the required [C] matrix of $\{\alpha\} = [C]\{w\}$ and then the local [B] bending matrix as: $\underbrace{[B^b]}_{3x12} = \underbrace{[R]}_{3x6} \underbrace{[H]}_{6x12} \underbrace{[C]}_{12x12}$. The one point quadrature is achieved at the centred local frame for the membrane strains using the perturbation or the physical stabilisation procedures and the necessary warping corrections.

4 NUMERICAL EXAMPLES

4.1 Springback analysis:

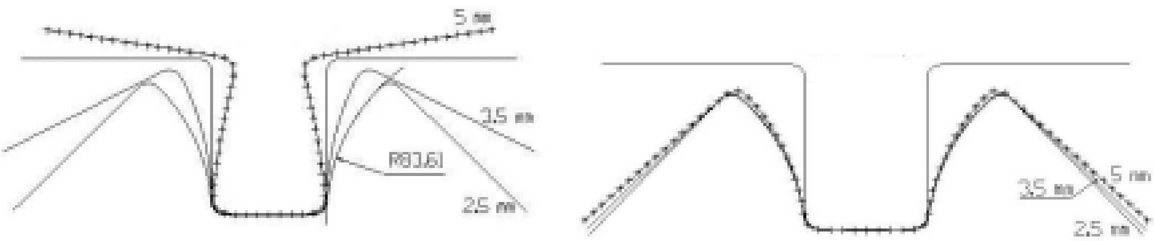

Figure 3: Mesh size sensitivity with conventional assumed stain element on the left and with rotation-free shell elements on the right.

This U-bending test was used as a benchmark problem for NUMISHEET'93 [2] to compare various FEM codes. On Fig. (3), the example shows the great mesh size sensitivity of conventional assumed strain 4-node shell elements which is not the case for the rotation-free elements. The mesh size sensitivity is mainly due to the *rotational d.o.f.* because they are not controlled by the contact algorithms.

4.2 Crash of a box beam:

The rotation-free element "S3" has been implemented in the commercial explicit code *"RADIOSS"* of MECALOG (France) under the name "S3N6" and has been compared to two other 3-node triangles: "DKT18" for Discrete Kirchhoff Triangle and "T3C0" for the Belytschko-Tsay triangular element. A box beam of rectangular cross section is impacted by an infinite mass with a constant velocity of 1.27 m/s:

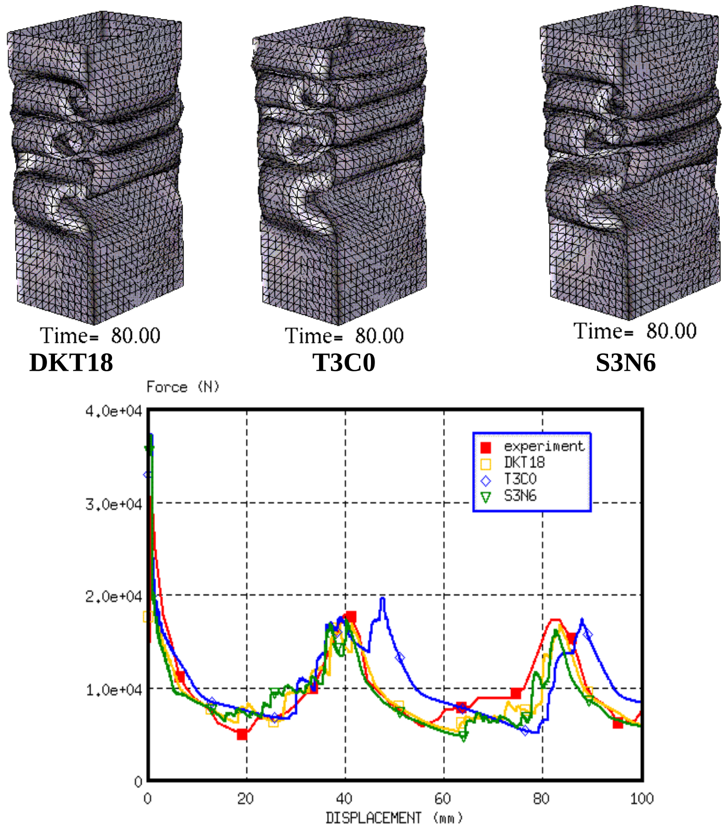

Figure 4: Normal force in the impacting rigid-wall versus displacement (3x3 mm mesh size)

REFERENCES

[1] Oñate E, Zarate F. "Rotation-free triangular plate and shell elements" *International Journal for Numerical Methods in Engineering* **47**, 557-603 (2000).

[2] Brunet M, Sabourin F. "A simplified triangular shell element with necking criterion for 3D. sheet forming analysis" in *NUMISHEET'93* edited by Makinouchi A, Nakamachi E, Oñate E, Wagoner RH, Riken, Tokyo 1993; pp. 229-238.

[3] Sabourin F, Brunet M. "Analysis of plates and shells with a simplified three-node triangular element" *Thin Walled Structures* **21**, 209-223 (1995).

[4] Sabourin F, Fayet M. "Incorporation of theory of screws in a new finite element" *Journal of Multibody Dynamic* **215**, 61-73 (2001).

[5] Sabourin F, Vives M. « Eléments finis triangulaires pour la simulation de l'emboutissage » *Revue Européenne des éléments finis* **10**, 7-53 (2001).