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USING THE WORST SCENARIO METHOD FOR ERROR AND SCATTER ESTIMATION IN ELASTO-PLASTIC ANALYSIS

Jiří Plešek*, **Ivan Hlaváček**†, **and Dušan Gabriel***

*Institute of Thermomechanics
Dolejškova 5, Prague 8, 182 00
Czech Republic
e-mail: plesek@it.cas.cz, web page: http://www.it.cas.cz/

†Mathematical Institute
Žitná 25, Prague 1, 115 67
Czech Republic
e-mail: hlavacek@math.cas.cz

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Summary. *Three models of elastoplasticity are compared on the basis of a classic example of a cyclically loaded notched bar. The numerical solution displays a monotonous dependence of the residual plastic strain, at the notch root, on the uncertain input material characteristics.*

1 INTRODUCTION

The recent treatise[1] deals in a broad context with the solution sensitivity to perturbed input conditions for a variety of engineering problems. Based on the earlier publications by the second author of this paper, the existence of a worst scenario was proven for three types of the criterion functional related to a small strain elastic-plastic continuum formulation, employing isotropic, kinematic and mixed hardening rules. Later on, Hlaváček, Plešek and Gabriel applied these material models to a typical problem of plasticity, a cyclically loaded notched bar, subject to variations of the yield stress, hardening modulus and the elastic parameters[2]. The influence of the input variation on the normal stress component in the plastic zone was investigated; with the main results also found in the above mentioned book. In this work the attention was focussed on a more sensitive and, perhaps, from the engineering view point even more important quantity, the maximum residual strain at the notch root.

2 WORST SCENARIO METHOD

One of the simplest non-probabilistic procedures is the method of *worst scenario* (WSM) alias *anti-optimization*. It consists in the three following steps: (i) choice of a criterion functional $\Phi(A, u(A))$ in accordance with the technical requirements, where

$u(A)$ denotes the solution of the mathematical model for input data A; (*ii*) definition of a set U_{ad} of admissible input data, and (*iii*) solving the maximization problem

$$\max_{A \in U_{ad}} \Phi(A, u(A)). \tag{1}$$

For an analysis of the method in various fields of physics and engineering we refer to monograph[1]. In the present paper we restrict ourselves to an illustrative example involving three well-known models of elastoplasticity, namely the perfect (ideal) plasticity and models with isotropic or kinematic hardening.

In papers[3,4,5] the existence of a worst scenario, i.e. a solution of problem (1), was proven for three types of the criterion functional. Approximate solutions based on the finite element discretization in space and backward differences in time were introduced in Ref.[3,4] together with some convergence analysis. The case of kinematic hardening was treated in Ref.[1]-Section 23. Another formulation of the model with isotropic hardening was analyzed in paper[5].

Here we will display numerical solutions of problem (1) for the classic plane-stress example of a cyclically loaded notched inelastic bar. The numerical solution of problem (1) consists heavily in the sensitivity analysis, i.e. in the analysis of properties of the mapping

$$A \mapsto \Phi(A, u(A)) \,\text{for}\, A \in U_{ad}. \tag{2}$$

If this mapping happens to be monotonous in some sense, the solution to problem (1) is obvious since the maximum is then attained on the boundary of the set U_{ad}. Fortunately, this will be the case for all the three plasticity models under consideration.

3 INCREMENTAL CONSTITUTIVE MODELS

Details of the mixed hardening elastoplastic model as it is implemented in the finite element package PMD and used in this study are presented in Ref.[6]. For the current purposes the model is specialized to von Mises' linear hardening J_2-theory.

Throughout the text, the usual notation is used: $\boldsymbol{\sigma}$, $\mathbf{S}$ and $\mathbf{h}$ denote the stress tensor, its deviatoric part and the backstress tensor, respectively; $\boldsymbol{\epsilon}$, $\boldsymbol{\epsilon}^p$ are the total strain and the plastic strain tensors; H is the hardening modulus $H = (E - E_t)/EE_t$, where E is the Young modulus and E_t is the slope of the uniaxial stress-strain curve; and λ, G are the Lamé constants. Dots superimposed over the characters denote (material) time derivatives while the double dot symbol means the inner product of two second-order tensors, e.g., $\mathbf{S} : \mathbf{S} = S_{ij}S_{ij}$. All the computations were performed within small strain theory.

The von Mises effective stress with the inclusion of backstress is defined as

$$\sigma_e = \sqrt{\tfrac{3}{2}(\mathbf{S} - \mathbf{h}) : (\mathbf{S} - \mathbf{h})} \tag{3}$$

The yield condition

$$\sigma_e = \bar{\sigma}_Y(\epsilon_p) \tag{4}$$

permits both isotropic and kinematic hardening, making the subsequent yield stress $\bar{\sigma}_Y$ dependent on equivalent plastic strain ϵ_p whose increment is given by

$$\dot{\epsilon}_p = \sqrt{\tfrac{2}{3}\,\dot{\boldsymbol{\epsilon}}^p : \dot{\boldsymbol{\epsilon}}^p} \tag{5}$$

For perfectly plastic or kinematic hardening model the initial yield stress σ_Y must be substituted, i.e. $\bar{\sigma}_Y = \sigma_Y = const.$

Incremental constitutive equations describing the evolution of kinematic variables can be summarized as follows.

$$\dot{\boldsymbol{\epsilon}}^p = \frac{3}{2}\frac{\dot{\epsilon}_p}{\sigma_e}(\mathbf{S} - \mathbf{h}) \tag{6}$$

$$\dot{\mathbf{h}} = \dot{\mu}(\mathbf{S} - \mathbf{h}) \tag{7}$$

$$\dot{\epsilon}_p = \frac{3G}{H + 3G}\,\frac{\mathbf{S} : \dot{\boldsymbol{\epsilon}}}{\sigma_e} \tag{8}$$

$$\dot{\mu} = H\frac{\dot{\epsilon}_p}{\sigma_e} \tag{9}$$

Note, that in the J_2-theory the Prager and Ziegler hardening rules coincide.

Finally, having plastic strain computed, the stress tensor is determined from the generalized Hooke's law as

$$\boldsymbol{\sigma} = \lambda \mathrm{tr}(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^p)\mathbf{I} + 2G(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^p). \tag{10}$$

The tangent stiffness-radial corrector scheme is employed to integrate the evolution equations (6)–(9). In this algorithm, time derivatives are replaced with small increments as in the Euler forward method, followed by the radial return correction performed on the stress tensor components only so that the yield condition (4) is exactly satisfied. The method possesses unconditional numerical stability and convergence for any convex yield criterion. Refer to paper[7] for details.

4 TEST PROBLEM

A flat bisymmetric notched inelastic bar, whose geometry, material properties, and pulse loading data were given in Ref.[2,6], was analyzed for three considered models of elastoplasticity: perfect plasticity, isotropic hardening and kinematic hardening models. The estimated discretization error of numerical analyses (less than 1%) was significantly smaller than the scatter of experimental data (about 10%).

When the average stress in the plastic zone was taken as a WSM criterial function, the results were little sensitive to the input data variation[2]. Much better sensitivity was achieved by detection of the residual plastic strain measured by the strain gauge placed directly at the root of the notch. The main points of this analysis and some general observartions are summarized below.

5 CONCLUSIONS

- As in the paper[2] for a stress indicator, the sensitivity mapping for residual plastic strain turned out to be monotonous so that the maximum of the criterial quantity was attained on the boundary of the admissible set.
- Comparing to the earlier analysis, this admissible set had to be substantially enlarged to contain the results based on the new criterion. More specifically, much larger interval of the input yield stress variation had to be considered.
- The present numerical experiments show, on the one hand, that WSM must be used with care, bearing in mind which quantity sensitivity has been studied. On the other hand, WSM may act as a powerful tool to estimate the validity and scope of the constitutive model used. For example, one might readily simplify analysis, resorting to less demanding material models if the error/scatter predicted by WSM can still be considered acceptable.

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