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A STUDY ON KINEMATIC HARDENING MODELS FOR THE SIMULATION OF CYCLIC LOADING IN FINITE ELASTO-PLASTICITY BASED ON A SUBSTRUCTURE APPROACH

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Summary. *Cyclic loading processes are characterized by a hardening behavior distinctly differing from that one obtained in uniform loading. The specific stress response is essentially influenced by various process parameters. Difficulties in numerical modeling of real cyclic forming processes arise because of the varying load history obtained in different material points. This paper presents some basic relations of a material law for finite elasto-plasticity founded on a substructure concept. Scalar and tensorial internal variables are defined to consider isotropic as well as anisotropic (kinematic, distorsional) hardening behavior. Within this context, special attention is turned to the effect of different approaches for the evolution of kinematic hardening on the cyclic stress response.*

1 INTRODUCTION

Due to the complexity of the technological and mechanical processes during metal forming, local stress analyses require the application of suitable numerical approaches like the FEM. A special class of real forming processes, the so-called incremental forming procedures (e. g. spin extrusion), are characterized by cyclic loading of the workpiece.

A sufficiently accurate numerical modeling of the material behavior requires the consideration of suitable material laws. Within this context, for the numerical simulation of real cyclic processes large elasto-plastic strains as well as anisotropic hardening behavior have to be considered. Furthermore, in dependence on cycle parameters, experiments on cyclic torsion show hardening effects different from those at monotonic loading.

We developed a thermodynamically consistent material law for large elasto-plastic deformations based on a substructure approach. Within this context, defining evolutional equations for special scalar and tensor-valued internal variables it is capable to describe isotropic as well as kinematic and distorsional hardening.

The selection of appropriate evolutional equations for kinematic and distorsional hardening is essentially for capturing the typical cyclic effects. On the example of the kinematic hardening the study shows the influence of several approaches like Prager type or Armstrong-Frederick type evolutional equations on the numerical results.

The substructure model has been implemented into the non-commercial adaptive FE-code SPC-PM2AdNl developed at the Chemnitz University of Technology. A numerical example shows its capability to predict the stress response in cyclic loading.

2 MATERIAL LAW FOR ANISOTROPIC FINITE ELASTO-PLASTICITY BASED ON A SUBSTRUCTURE APPROACH

Following the majority of recent investigations, the modeling of the kinematics of finite elasto-plastic deformations is based on the multiplicative split of the deformation gradient $\boldsymbol{F} = \boldsymbol{F}^e \boldsymbol{F}^p$ into an elastic part $\boldsymbol{F}^e$ and a plastic part $\boldsymbol{F}^p$. Within this context, three different configurations related to the continuum are considered – the reference configuration, the current configuration, and the plastic intermediate configuration.

The crucial point of the presented elasto-plastic material model is the definition of a so-called substructure configuration (with the metric tensor $\widehat{\boldsymbol{G}}$) which differs from the plastic intermediate configuration by rotations. Internal variables describing the hardening behavior are basically defined with respect to the substructure configuration, and transferred into the reference configuration with a mapping tensor $\boldsymbol{H}$.

Based on an additive split of the free Helmholtz energy density, and analyzing the Clausius-Duhem inequality a thermodynamically consistent material model describing anisotropic finite elasto-plastic deformations can be defined as a system of differential and algebraic equations. For details we refer to Bucher et al.[1]

Modifying appropriately a von Mises type approach the following substructure-based yield condition F is suggested:

$$F = \left(\grave{\boldsymbol{T}} - \grave{\boldsymbol{\alpha}}\right) \boldsymbol{C} \cdot\cdot \left(\grave{\boldsymbol{T}} - \grave{\boldsymbol{\alpha}}\right) \boldsymbol{C} + c_s \boldsymbol{M}_S \boldsymbol{C} \cdot\cdot \boldsymbol{M}_S \boldsymbol{C} - \frac{2}{3} T_F^2 = 0 \tag{1}$$

$$\text{with} \qquad \boldsymbol{M}_S = \boldsymbol{\alpha} \boldsymbol{C} \left(\boldsymbol{T} + \boldsymbol{T}^p\right) + \left(\boldsymbol{T} - \boldsymbol{T}^p\right) \boldsymbol{C} \boldsymbol{\alpha} \tag{2}$$

$$\text{and} \qquad \grave{\boldsymbol{T}} = \boldsymbol{T} - \frac{1}{3} \left(\boldsymbol{T} \cdot\cdot \boldsymbol{C}\right) \boldsymbol{C}^{-1} \qquad (\grave{\boldsymbol{\alpha}} \text{ analogously}) . \tag{3}$$

Here $\boldsymbol{C}$ denotes the right Cauchy-Green tensor, $\boldsymbol{T}$ the 2nd Piola-Kirchhoff stress tensor, $\boldsymbol{\alpha}$ the backstress tensor, and $\boldsymbol{T}^p$ a skew-symmetric stress-like tensor.

For the evolution of the yield stress T_F in dependence on the plastic arc length E_v^p the power law

$$T_F = T_{Fo} + a \left[\left(E_v^p + \beta\right)^n - \beta^n \right] \tag{4}$$

is used. Furthermore, the evolutional equation

$$\dot{\boldsymbol{\alpha}} = -c_{11} \boldsymbol{X} \left(\lambda \frac{\partial F}{\partial \boldsymbol{\alpha}}\right) \boldsymbol{X} - \boldsymbol{H}^{-1} \dot{\boldsymbol{H}} \boldsymbol{\alpha} - \boldsymbol{\alpha} \dot{\boldsymbol{H}}^T \boldsymbol{H}^{-T} \quad \text{with} \quad \boldsymbol{X} = \boldsymbol{H}^{-1} \widehat{\boldsymbol{G}} \boldsymbol{H}^{-T} \tag{5}$$

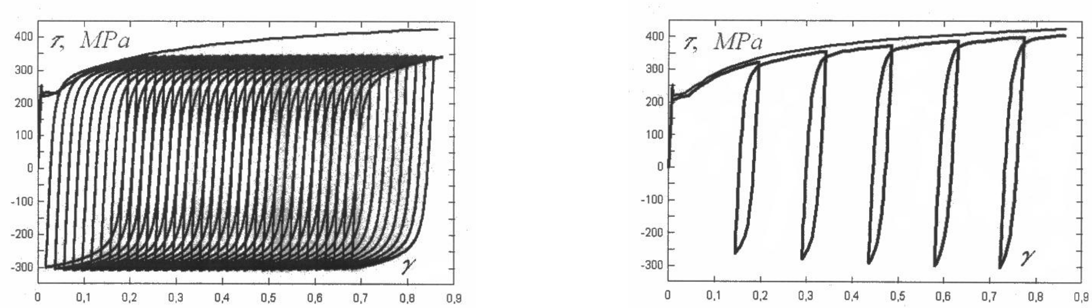

Figure 1: Stress-strain curves of cyclic torsion of thin-walled cylindrical 20MoCrS4 steel samples. Forming step width $\Delta\gamma_+ = 0.191$. Forming increment $\Delta\gamma_i = 0.017$ (left), $\Delta\gamma_i = 0.139$ (right).

for the back stress $\boldsymbol{\alpha}$ with the plastic multiplier λ and an analogous relation for $\boldsymbol{T}^p$ (with parameter c_{21} instead of c_{11}) are defined strictly following thermodynamical restrictions.

For an improved modeling of the Bauschinger effect Armstrong and Frederick[2] enhanced Prager type relations for the evolution of the back stress tensor adding a relaxation term. Based on some recent publications[3–5] we propose the following Armstrong-Frederick type evolutional equation considering a substructure:

$$\dot{\boldsymbol{\alpha}} = -c_{11}\boldsymbol{X}\left(\lambda\frac{\partial F}{\partial \boldsymbol{\alpha}}\right)\boldsymbol{X} - \boldsymbol{H}^{-1}\dot{\boldsymbol{H}}\,\boldsymbol{\alpha} - \boldsymbol{\alpha}\,\dot{\boldsymbol{H}}^T\boldsymbol{H}^{-T} - c_{12}\,\lambda\,\boldsymbol{\alpha} \tag{6}$$

3 NUMERICAL EXAMPLE

Based on extensive experimental investigations Meyer et al.[6] present some typical results of cyclic loading of samples of the steel alloy 20MoCrS4 under various loading regimes.

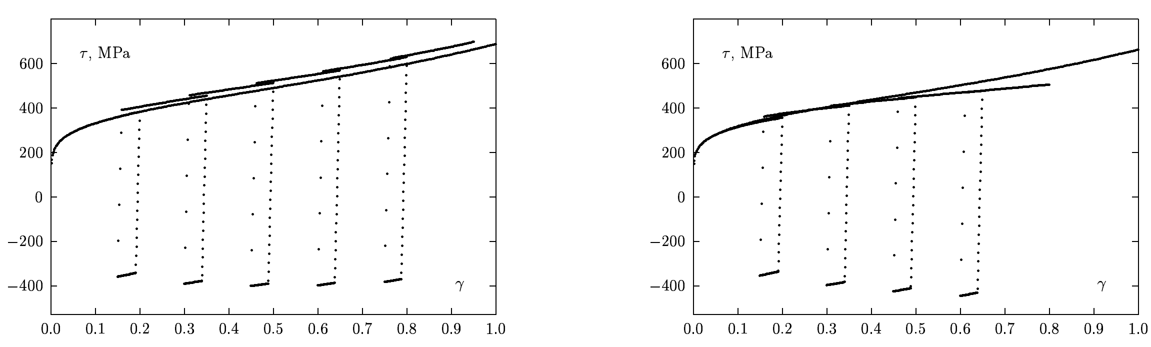

Figure 2: Numerical simulation of the cyclic simple shear. Forming step width $\Delta\gamma_+ = 0.2$, and forming increment $\Delta\gamma_i = 0.14$. Left: Prager type model (5). Right: Armstrong-Frederick type model (6). Material parameters: $T_{Fo} = 200\,\mathrm{MPa}$, $a = 700\,\mathrm{MPa}$, $\beta = 10^{-8}$, $n = 0.25$ (4); $c_{11} = 200$ (5,6); $c_{12} = 10^5$ (6); $c_s = 0.01\,\mathrm{MPa}^{-2}$ (1); $c_{21} = 100$.

Exemplarily, Fig. 1 shows the difference between cyclic hardening and uniform loading hardening response depending on the forming increment in the case of a constant forming step width.

On the example of the simple shear of a quadratic sheet differences in cyclic stress response between several rules for kinematic hardening within the context of the substructure concept could be demonstrated. If in the case of the Prager type model (5) stresses obtained for cyclic loading are higher than for the uniform case, the implementation of the Armstrong-Frederick type model (6) improved the result of numerical simulations compared with the experiments (see Fig. 2).

4 CONCLUSIONS

In this paper we presented a material model for finite elasto-plasticity considering a general plastic anisotropy. It is based on the idea of associating a macroscopic phenomenological approach with real material structures on micro-scale level motivated by the separation of the kinematics of the continuum and the kinematics of the substructure.

An appropriate yield condition and evolutional equations for the internal variables are presented. The material model has been implemented into a non-commercial adaptive finite element code developed at the Chemnitz University of Technology, and can successfully be applied for the numerical simulation of cyclic stress response.

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