VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)

CONSISTENT TIME-CYCLE APPROXIMATION FOR CYCLIC PLASTICITY COUPLED TO DAMAGE

Fredrik Larsson⋆ and Kenneth Runesson⋆

⋆Department of Applied Mechanics
Chalmers University of Technology
SE-41296 Göteborg, Sweden
e-mail: fredrik.larsson@sem.chalmers.se, web page: http://www.chalmers.se

Key words: Cyclic loading, Damage.

Summary. *This contribution deals with a solution strategy pertinent to cyclic loading for a large number of cycles. Rather than integrating the problem for the entire time-domain, a multi-scale time-cycle-modeling approach is introduced. The strategy is applied to a constitutive model involving damage coupled to plasticity.*

1 INTRODUCTION

Simulating cyclic loading of materials using conventional constitutive models in the time-domain is inherently expensive when it is desirable to trace the response under a large number of cycles. One way of circumventing this drawback is to introduce a coordinate transformation and substitute the cycle number for the physical time as the independent variable. Such an approach, however, requires that the evolution laws of the constitutive model are derived in terms of, e.g., amplitudes and mean values rather than the "actual" state variables used in a conventional constitutive model.

In this contribution, we propose a novel method of constructing consistent cyclic modeling based on conventional time-domain-models. The method relies on an *exact* difference equation in terms of individual cycles, where each time-cycle constitutes one discrete cycle step. Taking the exact difference equation as the starting point, we device a FE approximation in the cycle-domain, corresponding to the Quasi-Continuum Method in a spatial domain, whereby the difference equation is solved approximately. The resulting strategy, where the actual time history need be integrated for a few individual cycles only, allows for global error control. An adaptive algorithm, based on duality arguments, is then applied to adapt the finite element mesh in the cycle-domain with respect to a chosen error tolerance. In this manner, we resolve a minimal number of actual time-cycles in order to reach a prescribed accuracy.

This paper gives the theoretical basis for the time-cycle approximation in terms of formulating the exact difference equation (in the cycle-domain) pertinent to a canonical time-dependent problem. Furthermore, we introduce the approximate solution of the cycle-domain problem.

2 THE DIFFERENCE EQUATION

Consider the canonical 1st order problem in time over the time-interval $I = (t_0, T)$,

$$\begin{array}{ll} \mathrm{d}_t x + A(x) = f(t) & t \in I = (t_0, T) \\ x = x_0 & \text{at } t = t_0 \end{array} , \tag{1}$$

where x represents a (discrete) set of states, d_t indicates the time derivative and $A(x)$ represents the time-invariant system properties. x_0 represents initial data and $f(t)$ embodies the external loading.

Assume that a numerical solution of (1) becomes extremely expensive to compute due to a large range of timescales; the duration of the simulation time, the varying loading, and the system properties. In such cases, we wish to find a "smoother" solution x in terms of a *macro-scale* response, i.e., the solution described on the time-scale of the simulation time. To this end, we subdivide the time-interval into sub-increments, or "cycles", $I = \bigcup_{n=1}^{N} I_n$, where $I_n = (t_{n-1}, t_n)$ is a cycle of individual length $k_n = |I_n| = t_n - t_{n-1}$. The choice of cycles should relate to a "near-periodic" solution of x for the proposed algorithm to work efficiently, i.e., the cycles should relate to the properties of $f(t)$ (near stationary solutions) and/or $A(x)$ (near homogeneous (harmonic) solutions). However, we stress that the framework is consistent and thus, independently of the choice of cycles, (i) the solution converges with increasing number of cycles (ii) the framework allows for global error control.

We now define the macro-scale solution to be $X_n = x(t_n)$, i.e., X_{n-1} represents the initial value for the problem at hand on the interval I_n. The canonical problem (1) can now be reformulated into an *exact* difference equation as follows:

$$\begin{array}{ll} X_n = X_{n-1} + \Delta X(X_{n-1}, n) & n = 1, 2, \ldots, N \\ X_0 = x_0 & \end{array} , \tag{2}$$

where the difference term ΔX is defined through a *local problem*. In order to establish the local problem, we introduce the local solution on cycle n, ξ_n, and a local time, $\tau = (t - t_{n-1})/k_n \in (0, 1)$, whereby $\Delta X(X_{n-1}, n) = \xi_n(1) - \xi_n(0)$ and $\xi_n(\tau)$ is the solution of

$$\begin{array}{ll} \mathrm{d}_\tau \xi_n + k_n A(\xi_n) = \hat{f}_n(\tau) & \tau \in (0, 1) \\ \xi_n = X_{n-1} & \text{at } \tau = 0 \end{array} , \tag{3}$$

where

$$\hat{f}_n(\tau) \stackrel{\text{def}}{=} k_n f(t_{n-1} + \tau k_n). \tag{4}$$

It can easily be verified that (2) and (3) give the identical result as (1), i.e.,

$$\xi_n(\tau) \equiv x(t_{n-1} + \tau k_n) \quad \text{for } \tau \in (0, 1) \text{ and } n = 1, 2, \ldots, N. \tag{5}$$

The purpose of introducing (2) and (3) is to obtain good results with (2) approximated sparsely, i.e., we hope that $\{X_n\}$ is a piecewise smooth sequence such that ΔX can be

approximated accurately without actually solving (3) for all cycles n. Hence, as discussed earlier, any solution to (2) and (3) is an exact solution to (1), independent of the choice of $\{I_n\}$, whereas a good choice of cycles allows for good approximation properties of (2).

The exact solution sequence to (2) is denoted $Z = \{X_n\}_{n=0}^{N}$. We are thus in the position to formulate the variational, or weak, form of (2) as follows: Find $Z \in \mathcal{X}$ such that

$$a(Z, \delta Z) = 0 \quad \forall \delta Z \in \mathcal{X}, \tag{6}$$

where $\mathcal{X}$ is the space of all admissible sequences. In standard, Galerkin, fashion, we now introduce a reduced-dimensional space $\mathcal{X}_R$ corresponding to the Quasi-Continuum method, whereby the approximate problem reads: Find $Z_R \in \mathcal{X}_R$ such that

$$a(Z_R, \delta Z_R) = 0 \quad \forall \delta Z_R \in \mathcal{X}_R. \tag{7}$$

Solving (7) approximately, evaluating only a few cycles in Gauss-point fashion, is generally of much less cost than that of solving the "exact" problem (1). Using the technique of solving a dual problem, we may estimate the error $Z - Z_R$ due to the approximate space $\mathcal{X}_R \subset \mathcal{X}$. Note that even the space $\mathcal{X}$ is finite, dim $\mathcal{X} = N$. Increasing the dimension of $\mathcal{X}_R$ until $\mathcal{X}_R = \mathcal{X}$ results in the resolution of each cycle, and thus gives the exact solution to (1). In conjunction with an adaptive algorithm, the error estimate thus allows for balancing of computational effort (number of resolved cycles) and accuracy.

As the model problem, we wish to study the case of a material point (homogeneous specimen) subjected to cyclic loading, whereby the evolution of damage coupled to plastic deformation is investigated using the proposed method. This numerical investigation will be presented at the conference.