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A GENERALIZED BACKWARD EULER SCHEME FOR THE INTEGRATION OF A MIXED ISOTROPIC-KINEMATIC HARDENING MODEL FOR CLAYS

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1 INTRODUCTION

The mechanical response of intact clays is characterised by highly non-linear behaviour, memory of the past strain-history, evolving anisotropy, non-coaxiality and, when cemented, mechanically induced bond degradation phenomena.

In recent years a number of constitutive models have been proposed to mathematically describe these features, often being characterised by complex formulations leading to non-trivial problems in their numerical integration. On the other hand, accuracy and stability are recognised as crucial requirements in the development of any integration algorithm for realistic material models, in order to ensure the necessary computational correctness and efficiency in their use within Finite Element codes.

This paper describes a fully implicit stress-point algorithm for the numerical integration of a single surface mixed isotropic-kinematic hardening plasticity model for bonded clays. In the following the soil mechanics sign convention is assumed and all stresses are effective stresses.

2 CONSTITUTIVE MODEL

The constitutive model is formulated in the framework of classical rate-independent plasticity. The reversible behaviour is described by a hyperelastic formulation originally proposed by[1] and modified by[2], to include the elastic stiffness dependence on effective stresses; the elastic strain energy function is the following:

$$\Psi\left(\varepsilon_v^e,\varepsilon_s^e\right)=p_0\tilde{\kappa}\exp\left(\frac{\varepsilon_v^e-\varepsilon_{v0}^e}{\tilde{\kappa}}\right)+\frac{3}{2}\left[\mu_0+\alpha^* p_0\exp\left(\frac{\varepsilon_v^e-\varepsilon_{v0}^e}{\tilde{\kappa}}\right)\right]\left(\varepsilon_s^e\right)^2 \quad (1)$$

where ε_{v0}^e is the elastic volumetric strain corresponding to the mean stress p_0, which is here set = 0 for p_0=1 kPa, $\tilde{k}$ is the elastic compressibility index and μ_0 and α^* are shear and coupling parameters.

The elastic domain is defined by the convex set $\mathbf{E}_\sigma = \{(\boldsymbol{\sigma},\mathbf{q}) | f(\boldsymbol{\sigma},\mathbf{q}) \le 0\}$, where $\mathbf{q} = \mathbf{q}(\alpha, \boldsymbol{\sigma}_K)$ is a set of internal variables defining the position and the dimension of the yield function *f*:

$$f(\boldsymbol{\sigma},\alpha,\boldsymbol{\sigma}_K) = \frac{1}{c^2}(\mathbf{s}-\mathbf{s}_K):(\mathbf{s}-\mathbf{s}_K) + (p-p_K)^2 - \alpha^2 \tag{2}$$

The geometrical representation of the yield surface in the stress space $\boldsymbol{\sigma} \equiv (p,\mathbf{s})$ is an ellipsoid centred at point K with co-ordinates $\boldsymbol{\sigma}_K = p_K\mathbf{I} + \mathbf{s}_K$. The assumed flow rule is associated.

The isotropic hardening rule controls the size of the yield surface while the kinematic hardening governs the motion of the yield surface in the stress space. The former is a modified version of that originally proposed by[3] in their Model for Structured Soils:

$$\dot{\alpha} = \alpha\left[\frac{1}{\tilde{\lambda}-\tilde{\kappa}}\dot{\varepsilon}_v^p - \xi_v \exp\left(-\eta_v\varepsilon_v^d\right)\dot{\varepsilon}_v^d - \xi_s \exp\left(-\eta_s\varepsilon_s^p\right)\dot{\varepsilon}_s^p\right] = \dot{\gamma}h \tag{3}$$

It is composed by two volumetric terms and a deviatoric one. The first volumetric term is similar to the standard Modified Cam-Clay hardening law, while the following terms account for the volumetric and deviatoric strain induced structure degradation (debonding) by means of two separate exponential damage-type form. The volumetric strain induced destructuring is controlled by:

$$\varepsilon_v^d = \int_0^t \left|\dot{\varepsilon}_v^p\right| dt \tag{4}$$

During plastic deformation the centre K of the yield surface moves as follows:

$$\dot{\boldsymbol{\sigma}}_K = \frac{\dot{\alpha}}{\alpha}\boldsymbol{\sigma}_K + \psi\frac{\dot{\alpha}}{\alpha}\left(\mathbf{s} - \chi\frac{p}{p_K}\mathbf{s}_K\right) \tag{5}$$

where χ and ψ are parameters. It is formulated in such a way that along radial stress paths the centre of the yield surface initially moves to then achieve a stabilised position, corresponding to the imposed direction of the path. Finally, the loading/unloading criterion is expressed by the Kuhn-Tucker complementary conditions $\dot{\gamma} \ge 0,\ f(\boldsymbol{\sigma},\alpha,\boldsymbol{\sigma}_K) \le 0,\ \dot{\gamma} f(\boldsymbol{\sigma},\alpha,\boldsymbol{\sigma}_K) = 0$, leading to the standard consistency condition for the plastic multiplier $\dot{\gamma}$: $\dot{\gamma}\dot{f}(\boldsymbol{\sigma},\alpha,\boldsymbol{\sigma}_K) = 0$.

3 IMPLICIT NUMERICAL INTEGRATION

A Generalised Backward Euler algorithm is formulated in the space of elastic strain and internal variables for a general time step Δt between $[t_n, t_{n+1}]$ over which an increment of total strain $\Delta\boldsymbol{\varepsilon}_{n+1} = \nabla^s(\Delta\mathbf{u}_{n+1})$ is assigned. Firstly, the elastic predictor problem is solved by freezing the plastic flow:

$$\boldsymbol{\varepsilon}_{n+1}^{e,trial} = \boldsymbol{\varepsilon}_{n}^{e} + \Delta\boldsymbol{\varepsilon}_{n+1},\ \boldsymbol{\varepsilon}_{n+1}^{p,trial} = \boldsymbol{\varepsilon}_{n}^{p},\ \alpha_{n+1}^{trial} = \alpha_{n},\ \boldsymbol{\sigma}_{K,n+1}^{trial} = \boldsymbol{\sigma}_{K,n},\ \boldsymbol{\sigma}_{n+1}^{trial} = \frac{\partial\psi\left(\boldsymbol{\varepsilon}_{n+1}^{e,trial}\right)}{\partial\boldsymbol{\varepsilon}^{e}} \tag{6a,b,c,d}$$

If the consistency condition is satisfied ($f_{n+1}^{trial}\left(\boldsymbol{\sigma}_{n+1}^{trial},\alpha_{n+1}^{trial},\boldsymbol{\sigma}_{K,n+1}^{trial}\right)\leq 0$) the process is declared elastic, otherwise a plastic correction is required. In this case a system of 14 non-linear equations:

$$\begin{aligned}
\boldsymbol{\varepsilon}_{n+1}^{e} &= \boldsymbol{\varepsilon}_{n+1}^{e,trial} - \Delta\gamma_{n+1}\partial_{\boldsymbol{\sigma}} f_{n+1} \\
f_{n+1} &:= f_{n+1}\left(\boldsymbol{\sigma}_{n+1},\alpha_{n+1},\boldsymbol{\sigma}_{K,n+1}\right) = 0 \\
\alpha_{n+1} &= \alpha_{n+1}^{trial} + \Delta\gamma_{n+1}h_{n+1} \\
\boldsymbol{\sigma}_{K,n+1} &= \left[\boldsymbol{\sigma}_{K,n+1}^{trial} + \psi\frac{\alpha_{n+1}-\alpha_{n}}{\alpha_{n+1}}\left(\mathbf{s}_{n+1} - \chi\frac{p_{n+1}}{p_{K,n+1}}\mathbf{s}_{K,n+1}\right)\right]\left(1-\frac{\alpha_{n+1}-\alpha_{n}}{\alpha_{n+1}}\right)^{-1}
\end{aligned} \tag{7a,b,c,d}$$

in the 14 unknowns $\left\{{\boldsymbol{\varepsilon}_{n+1}^{e}}^{T},\alpha_{n+1},{\boldsymbol{\sigma}_{K,n+1}}^{T},\Delta\gamma_{n+1}\right\}$ is solved iteratively by means of Newton's method.

In order to maintain the quadratic rate of asymptotic convergence that characterises the application of the iterative Newton's method at the global level in a FE code, the consistent elasto-plastic tangent modulus has to be defined accordingly to the local integration algorithmic scheme. The described integration algorithm allows the consistent tangent operator to be computed in closed form. The following expressions is obtained in the elastic strain space:

$$\mathbf{D}_{n+1} := \frac{d\boldsymbol{\sigma}_{n+1}}{d\boldsymbol{\varepsilon}_{n+1}^{e}} = \frac{d\boldsymbol{\sigma}_{n+1}}{d\boldsymbol{\varepsilon}_{n+1}^{e,trial}} = \mathbf{D}_{n+1}^{e}\frac{d\boldsymbol{\varepsilon}_{n+1}^{e}}{d\boldsymbol{\varepsilon}_{n+1}^{e,trial}} \tag{8}$$

The accuracy, stability and convergence properties of the proposed algorithm are evaluated by numerical simulations of single element tests (c=0.816, μ_0=25000 kPa, α*= 100, $\tilde{\lambda}$=0.097, $\tilde{k}$=0.01). Figure 1 a,b shows the influence of the time step dimension on the results of an isochoric axisymmetric compression test in terms of stress path (deviatoric stress q – mean effective stress p) and stress strain curves (ξ_v= η_v= 0, ξ_s= 5, η_s= 10, ψ= χ=0). The corresponding quadratic convergence profiles are plotted in Figure 2. The satisfactory performance of the algorithm is confirmed by the iso-error maps shown in Figure 3 a,b, obtained by applying linear combinations of volumetric and deviatoric strain increments starting from the following initial conditions: p= 138 kPa, q= 92 kPa, α = 100 kPa and the yield locus centred on the isotropic axis. Figure 3a refers to deactivated destructuration and kinematic hardening parameters, while in 3b all the material parameters were activated (ξ_v= 10, η_v= 15, ξ_s= 5, η_s= 10, ψ= 0.6, χ= 1.0). In both set of analyses the error is remarkably low for a large part of the diagrams.

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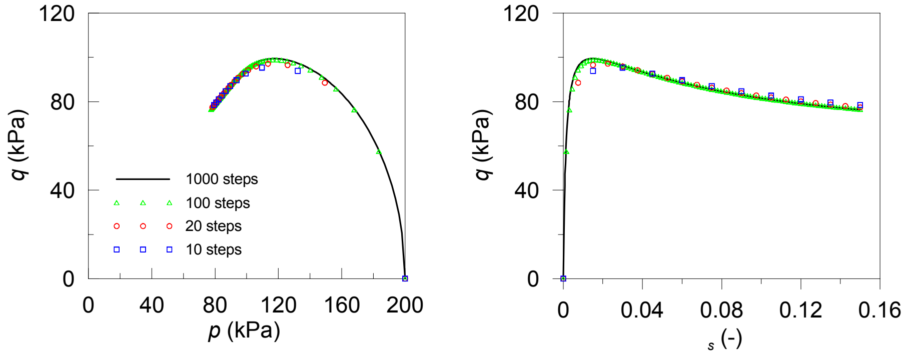

Figure 1a,b. Isochoric axisymmetric compression test: influence of the time step dimension.

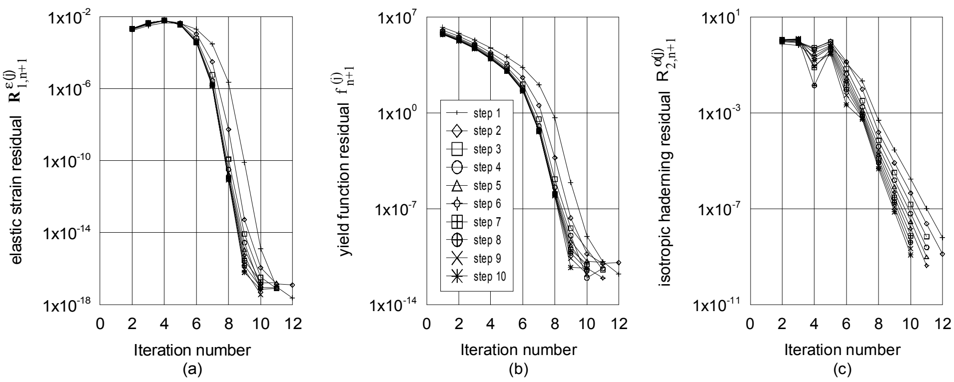

Figure 2. Convergence profiles from isochoric axisymmetric compression test.

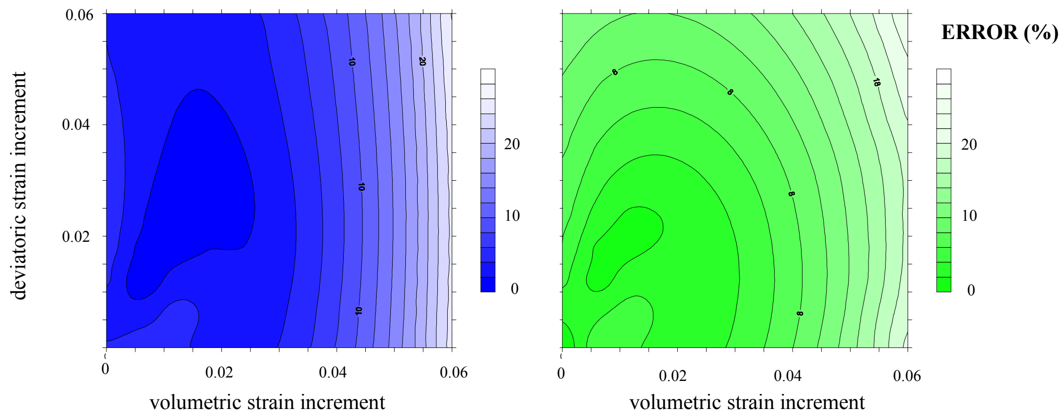

Figure 3a,b. Iso-error maps.