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A DISCRETE 2D QUASI-STATIC CYCLIC CONSTITUTIVE MODEL INCORPORATING HYSTERESIS FOR QUASI-BRITTLE MATERIALS.

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Summary. *A quasi-static constitutive model for a cracked material under mode I and II is presented in this extended abstract. This model incorporates the hysteretic phenomenon, which is observed during experimental loading. This model is subsequently adapted in a cohesive zone model, which is implemented in the partition of unity finite element method.*

1 INTRODUCTION

Cyclic quasi-static test results of quasi-brittle materials in the post-peak region[1,2], which corresponds with the cracked state of the material, reveal several different distinctive phenomena. So can one observe the reduction of the stiffness, the formation of permanent deformations and the formation of hysteretic loops during a loading-unloading cycle.

The reduction of the stiffness originates from the cracking of the material itself. The permanent deformations are caused by the imperfections of the cracks and debris between the crack faces, which precludes their complete closure upon unloading. During a cyclic loading of a test specimen in the post-peak region, the cracks are loaded and unloaded causing them to open and to close. During the opening and closing of the crack, the crack faces will slide over each other. Throughout this sliding, a friction takes place between the crack faces and an amount of energy is dissipated. This energy dissipation is clearly visible during a cyclic loading in the post-peak region[1,2] by the generation of hysteretic loops.

An accurate FEM-prediction of a cracked structure requires an accurate constitutive model. Consequently, it is preferable that this model would include all of the above-mentioned phenomena. In the remainder of the extended abstract, this model will be referred to as a combined damage-plasticity-hysteresis model. The generated model (combined damage-plasticity-hysteresis model) is subsequently discretised and implemented in FEM program based on the partition of unity to be able to simulate structures with discrete cracks.

2 THE COMBINED DAMAGE-PLASTICITY-HYSTERESIS MODEL.

2.1 Principle

The combined damage-plasticity-hysteresis model has been set up by the coupling of the combined damage-plasticity model as proposed by De Proft[5] with a scalar Preisach hysteresis model. The combined damage-plasticity model is capable of capturing all of the above-mentioned phenomena except for the observed hysteretic loops. Whereas the Preisach hysteresis model a very general model is to simulate hysteretic loops, it even includes the minor hysteresis loops in the major hysteresis loops. The corresponding energy dissipation of the hysteretic loops can be controlled by a characteristic model parameter that is called the Preisach density function. A coupling of these models can be obtained from a physical perspective. Before a coupling can be accomplished, a physical interpretation of the scalar Preisach hysteresis model, which is a general mathematical model[8], has to be provided. The Preisach hysteresis model can be cast in such a manner such that it describes the kinematics of the cracks, i.e. the opening and closing of the cracks during a cyclic loading. This kinematic action is accompanied with the sliding of the crack faces which causes the hysteretic phenomenon. Accordingly, it can be concluded that the hysteretic phenomenon is related with the number and size of the cracks. In its turn, the number and size of the cracks are coupled with the amount of damage. Consequently, it can be said that the combined damage-plasticity model and the Preisach hysteresis model can be coupled by the damage parameter.

2.2 Towards a 2D model.

As a consequence of the coupling of the combined damage-plasticity with a scalar Preisach hysteresis model can the hysteretic phenomena only be taken into account in a one dimensional manner. An extension (two dimensional) is obtained by splitting the displacements in components using a combined yield/fracture surface. From splitted components are subsequently the hysteretic parts calculated in the considered directions in an iterative manner. The surface used is a part of the surface proposed by Lourenço[7], i.e. a Mohr-Coulomb surface (mode II cracking) with a cut-off surface (mode I cracking), but transferred in the traction-separation space.

Because of microcracks in the material is the stress redistributed over the undamaged material. If one further assumes that the plastic deformation only occurs in the undamaged bonds. The stress working over these bonds, called the effective stress can then be written as:

$$\hat{T}_n = \frac{T_n}{1-d} \tag{1}$$

$$\hat{T}_t = \frac{T_t}{1-d} \tag{2}$$

Whereas the surfaces can be expressed in the effective space as:

$$f_p^R = \hat{T}_n - f_{t_o} \tag{3}$$

$$f_p^{MC} = \parallel \hat{T}_t \parallel + \hat{T}_n \cdot tan(\phi) - c_o \tag{4}$$

The plasticity model must be completed with a damage model. The damage in a material must grow when the damage loading function is violated:

$$f_d^R = \Delta_n - \kappa_n^{eq} \tag{5}$$

$$f_d^{MC} = \parallel \Delta_t \parallel + \Delta_n \cdot tan(\phi) - \kappa_t^{eq} \tag{6}$$

Equations (3-6) of the yield surfaces can be discretised by means of the kinematic equations over the crack.

2.3 Results of the model.

Results obtained by the material model are shown in figure (1) and (2). It is obvious that the constitutive model can take all of the above mentioned phenomena in account.

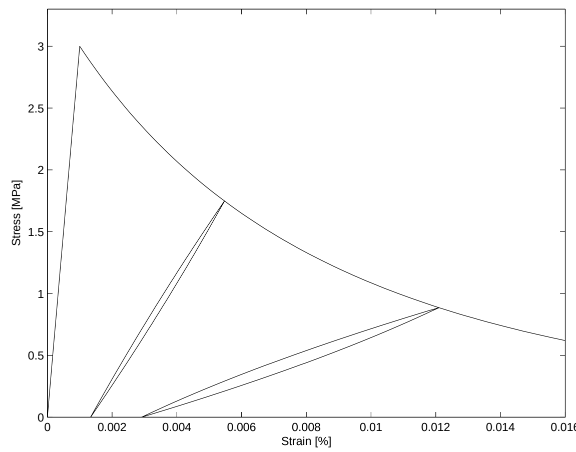

Figure 1: model's result in pure mode I(u_t=0)

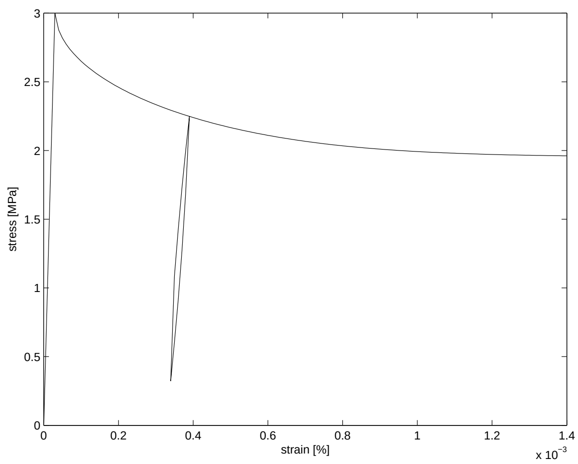

Figure 2: model's result in pure mode II(u_n=0)

3 CONCLUSIONS

A constitutive model is presented that allows to take all the observed phenomena (i.e. damage, plasticity, hysteresis) into account for mode I and mode II cracks.

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