VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)
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MICRO-MECHANICAL MODEL OF A VISCOPLASTIC OPEN-CELL FOAM

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Key words: Computational plasticity, Micro-mechanics, Open-cell foam, Large deformations.

1 INTRODUCTION

In applications where weight in relation to structural properties are critical in combination with energy absorption, structural foams have shown advantages in comparison to many other engineering materials. The objective of this study is to model the response of an open-cell foam at large deformations and high strain rates. The main idea from the micro-mechanical point of view is to consider the open-cell foam as a network of struts, where each strut connects two vertex points. The strut deformation is assumed to depend directly on the macroscopic deformation and the force carried by a strut are linked to the longitudinal change of its vertex-to-vertex vector. Thereby, the strut is modeled as a viscoplastic large deformation *1D* element. As a result, the microscopic Cauchy stress is established on the strut and further homogenized to the macroscopic Cauchy stress by averaging over a statistical ensemble of struts.

The model has been implemented in the context of large strains and viscoplastic Perzyna type behavior, with a dynamic yield surface.

2 DEFORMATION OF STRUT

2.1 Kinematics of strut

Consider the open-cell foam, in figure 1(a), represented as a network of struts, [1]. In particular, we focus on a representative strut in the network with the vertex points $\boldsymbol{X}_i'$ and $\boldsymbol{X}_i$ in the undeformed configuration, figure 1(b). Hence, the strut vertex-to-vertex vectors in the undeformed and deformed configuration are represented by; $\boldsymbol{R}_i$ and $\boldsymbol{r}_i$, respectively, defined as:

$$\boldsymbol{R}_i := \boldsymbol{X}_i' - \boldsymbol{X}_i = R_i \boldsymbol{N}_i \quad \text{and} \quad \boldsymbol{r}_i := \boldsymbol{x}_i' - \boldsymbol{x}_i = r_i \boldsymbol{n}_i, \tag{1}$$

where $\boldsymbol{N}_i$ and $\boldsymbol{n}_i$ are the Lagrangian and Eulerian directors of the strut;

$$\boldsymbol{N}_i := [\boldsymbol{X}_i' - \boldsymbol{X}_i] \parallel \boldsymbol{X}_i' - \boldsymbol{X}_i \parallel^{-1} \quad \text{and} \quad \boldsymbol{n}_i := [\boldsymbol{x}_i' - \boldsymbol{x}_i] \parallel \boldsymbol{x}_i' - \boldsymbol{x}_i \parallel^{-1}. \tag{2}$$

Assuming *Affine* deformation of the micro-structure i.e. $\boldsymbol{r}_i = \bar{\boldsymbol{F}} \cdot \boldsymbol{R}_i$, where $\bar{\boldsymbol{F}}$ is the macroscopic deformation gradient, we obtain the stretch $\boldsymbol{\lambda} := \boldsymbol{\lambda}_i = (r_i/R_i)\boldsymbol{n}_i$ and its magnitude

$$\lambda_i := \parallel \boldsymbol{\lambda}_i \parallel = (\boldsymbol{N}_i \cdot \bar{\boldsymbol{C}} \cdot \boldsymbol{N}_i)^{\frac{1}{2}}, \tag{3}$$

where λ is the total stretch of the strut and $\bar{\boldsymbol{C}} = \bar{\boldsymbol{F}}^T \bar{\boldsymbol{F}}$ is the *Right Cauchy-Green* deformation tensor.

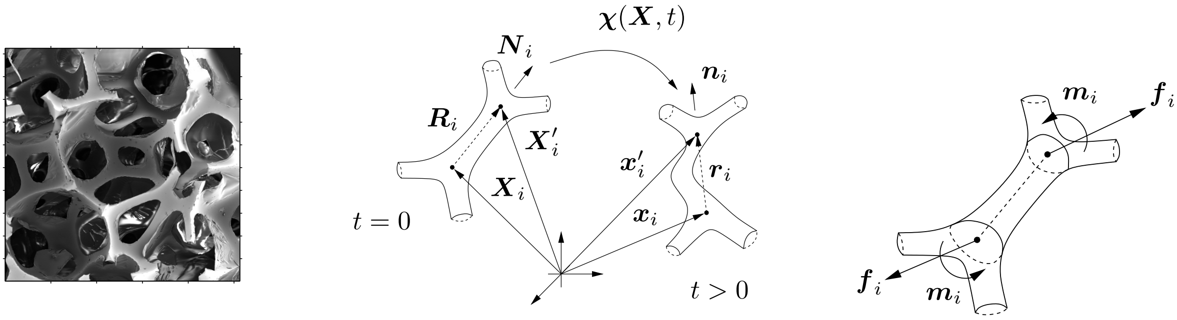
2.2 Constitutive modeling of longitudinal strut response - viscoplastic flow

In order to model the strong rate-dependence in the deformation of the foam micro structure, we assume that the stretch λ decomposes multiplicatively in a recoverable (elastic) component λ^{e} an irrecoverable (viscoplastic) component λ^{p} so that $\lambda = \lambda^{\mathrm{e}}\lambda^{\mathrm{p}}$, leading to the logarithmic strain, $\varepsilon := \log(\lambda)$, which is subdivided additively into an elastic and a plastic part, ε^{e} and ε^{p} written as

$$\varepsilon = \varepsilon^{\mathrm{e}} + \varepsilon^{\mathrm{p}} \quad \text{with} \quad \varepsilon^{\mathrm{e}} = \log(\lambda^{\mathrm{e}}) \quad \text{and} \quad \varepsilon^{\mathrm{p}} = \log(\lambda^{\mathrm{p}}). \tag{4}$$

For simplicity the strut force $\boldsymbol{f}$, see figure 1(c), is assumed to be parallel with the strut, so that $\boldsymbol{f}_i := f_i \boldsymbol{n}_i$, where the magnitude of the longitudinal component is assumed to possess the constitutive dependence $f_i = f_i(\varepsilon^{\mathrm{e}}, \kappa)$, where κ is the hardening variable associated with the micro-force $\mathrm{K} = \mathrm{H}\kappa$ and H is the hardening modulus. The resulting elastic law is formulated as linear elastic in the logarithmic strain; hence, the magnitude of the normal force is formulated in the stiffness coefficient (k) as

$$f^{\mathrm{e}} = \mathrm{k}\varepsilon_{\mathrm{e}} = \mathrm{k}(\varepsilon - \varepsilon_{\mathrm{p}}). \tag{5}$$



(a) Micro-structure of an open-cell foam.

(b) Deformation of strut.

(c) Forces and couples on strut.

Figure 1: Representation of the micro-stricture of an open-cell foam

To account for rate-dependent viscoplastic flow of the strut deformation, the inelastic component of the total strain evolves according to Perzyna's flow rule [3] as

$$\dot{\varepsilon}^{\mathrm{p}} := \frac{1}{t_*}\eta_{\mathrm{d}}(\xi)\frac{\partial\psi_{\mathrm{y}}}{\partial f} = \frac{1}{t_*}\eta_{\mathrm{d}}(\xi)\mathrm{sign}(f_i) \tag{6}$$

$$\dot{k} := \frac{1}{t_*}\eta_{\mathrm{d}}(\xi)\frac{\partial\psi_{\mathrm{y}}}{\partial \mathrm{K}} = -\frac{1}{t_*}\eta_{\mathrm{d}}(\xi), \tag{7}$$

where t_* is the relaxation time. Moreover, to model the decay of the rate-dependency at high strain rates and high stress levels, the over-stress function $\eta_{\mathrm{d}}(\xi)$ is formulated in the argument ξ as

$$\eta_{\mathrm{d}}(\xi) = \left(\frac{\xi}{1-\xi}\right)^{\mathrm{m}} \quad \text{with} \quad \xi = \frac{\langle\psi_{\mathrm{y}}\rangle}{\langle\psi_{\mathrm{y}}\rangle - \psi_{\mathrm{d}}}, \tag{8}$$

where m is the creep exponent, $\psi_{\mathrm{y}}(f,\mathrm{K})$ is the quasi-static yield function, whereas $\psi_{\mathrm{d}}(f)$ is the bounding or dynamic yield surface, defined as

$$\psi_{\mathrm{y}}(f,\mathrm{K}) := \mathrm{abs}(f) - f_{\mathrm{y}} - \mathrm{K} < 0 \quad \text{and} \quad \psi_{\mathrm{d}}(f) := \mathrm{abs}(f) - f_{\mathrm{d}} \quad \text{with} \quad f_{\mathrm{y}} \ll f_{\mathrm{d}}. \tag{9}$$

It may be noted that $\eta_{\mathrm{d}} = 0$ is obtained whenever $\psi_{\mathrm{y}} < 0$ corresponding to elastic behavior, whereas $\eta_{\mathrm{d}} \to \infty$ when the stress approaches the dynamic yield surface, i.e. when $\psi_{\mathrm{d}} \to 0$, corresponding to rate independent response.

2.3 Homogenization

The macroscopic Cauchy stress is obtained by averaging the dyadic $\boldsymbol{\sigma}_i = \boldsymbol{r}_i \otimes \boldsymbol{f}_i$ over a population of M struts:

$$\bar{\boldsymbol{\sigma}} = \mathrm{n}\left(\frac{\sum_{i=1}^{\mathrm{M}} \boldsymbol{r}_i \otimes \boldsymbol{f}_i}{\mathrm{M}}\right), \tag{10}$$

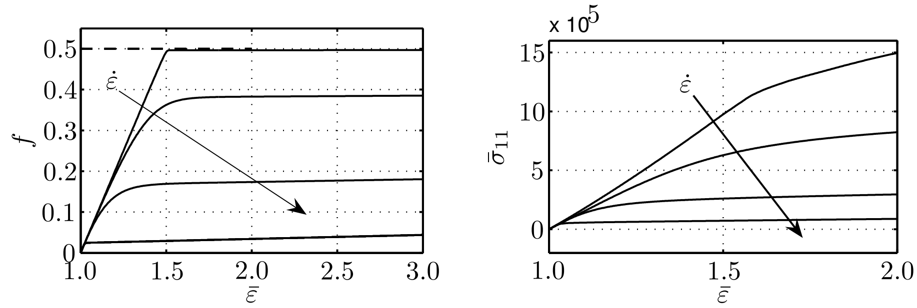
where n is the number fraction of struts in the material, cf. [4].

3 SIMULATIONS

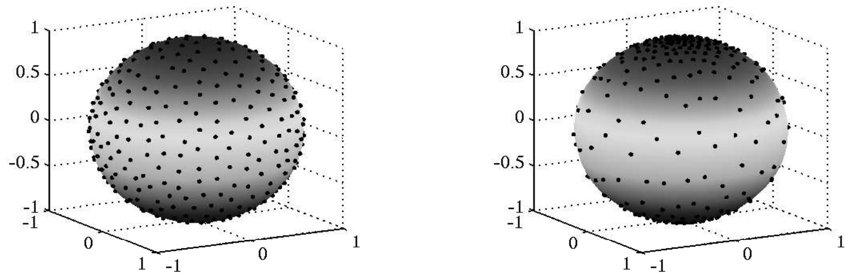
The proposed model has been implemented and preliminary results are obtained as shown in figure 2(a) and 2(b) for a simple tensile test with the macroscopic deformation considered as

$$\bar{\boldsymbol{F}} = \begin{pmatrix} \bar{\lambda} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with} \quad \dot{\bar{\lambda}} = 1\cdot 10^{-1}\ldots 1\cdot 10^{7}.$$

In figure 2(a) the force-strain response for a strut parallel to the loading direction is plotted. It is clearly seen that for the lower strain rates the static rate-independent elastic-plastic response situation is retrieved, whereas at the high strain rates the response is much stiffer and the response curves approach the dynamic yield surface (represented



(a) Microscopic force response on strut in in 11 direction. (b) Macroscopic stress-strain response.



(c) Lagrangian strut distribution. (d) Eulerian strut distribution.

by the dashed line ($f = f_{\mathrm{d}} = 0.5\mathrm{N}$) asymptotically as the strain rates are increased. The response in figure 2(b) shows the homogenized behavior for an (initially) isotropic network, as shown in figure 2(c) with a random distribution of strut orientations on the hemisphere. As expected, a smoother response for the network as compared to that of individual struts is obtained. In figure 2(d), the orientation of struts are depicted in terms of the corresponding deformed hemisphere, induced by the macroscopic deformation $\bar{\boldsymbol{F}}$.

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