VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)
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GRADIENT HARDENING AND DAMAGE IN CRYSTAL (VISCO)PLASTICITY

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Key words: Computational Plasticity, Crystal plasticity, Gradient plasticity.

1 INTRODUCTION

This contribution deals with the simulation of the behavior of a grain structure of a polycrystalline metal. The behavior of such a grain structure depends on, e.g., grain geometry, volume fraction of different phases, and grain size. A way of including the grain size dependence in the modelling of the grain structure will be presented. Alternative formulations can be found in, e.g., references[1,2].

Within the framework of continuum thermodynamics and finite strains, we formulate a model for crystal (visco)plasticity, crystal damage and gradient hardening. The crystal damage is based on the concept of a fictitious (undamaged) configuration, and it is assumed to be driven by inelastic slip in each slip system. Furthermore, the gradient hardening gives a contribution for each slip system which is added to the well established local hardening.

In order to solve the arising coupled field equations (for the displacements and the hardening of the slip systems) the dual mixed FE algorithm proposed in[3] is applied. The contribution is concluded by a numerical study how model parameters (such as grain size, damage rate) will influence the stress strain response of a 2D model of a polycrystal.

2 CRYSTAL PLASTICITY MODEL

In order to include a gradient dependence of the hardening variable k_α associated with the slip direction $\bar{\boldsymbol{s}}_\alpha$, we propose the following free energy (per unit undeformed volume):

$$\Psi(\bar{\boldsymbol{C}}, \bar{\boldsymbol{b}}_{\mathrm{d}}, k_\alpha, \bar{\boldsymbol{s}}_\alpha \cdot \nabla k_\alpha) = \Psi_{\mathrm{e}}(\bar{\boldsymbol{C}}, \bar{\boldsymbol{b}}_{\mathrm{d}}) + \sum_\alpha \left[\frac{1}{2} H_{\mathrm{l}} k_\alpha^2 + \frac{1}{2} H_{\mathrm{g}}\, l_\alpha^2 \left[\bar{\boldsymbol{s}}_\alpha \cdot \nabla k_\alpha \right]^2 \right] \tag{1}$$

where $\bar{\boldsymbol{C}}$ is the elastic Cauchy-Green tensor and $\boldsymbol{b}_{\mathrm{d}}$ is the integrity tensor that models the crystal damage. For details concerning the crystal damage we refer to Ekh et al.[4].

The plastic part of the free energy can now be used to define (by following the arguments put forward by Svedberg & Runesson[5]) the local hardening stress $\kappa_{\alpha,\mathrm{l}}$ and the gradient hardening stress $\kappa_{\alpha,\mathrm{g}}$:

$$\kappa_{\alpha,\mathrm{l}} = -\frac{\partial \Psi}{\partial k_\alpha} = -H_\mathrm{l}\, k_\alpha \tag{2}$$

$$\kappa_{\alpha,\mathrm{g}} = \nabla \cdot \frac{\partial \Psi}{\partial \nabla k_\alpha} = H_\mathrm{g}\, l_\alpha^2\, \frac{\partial^2 k_\alpha}{\partial \bar{s}_\alpha^2} \tag{3}$$

Hence, the total hardening κ_α in each slip system consists of a local and a gradient part:

$$\kappa_\alpha = \kappa_{\alpha,\mathrm{l}} + \kappa_{\alpha,\mathrm{g}} = -H_\mathrm{l}\, k_\alpha + H_\mathrm{g}\, l_\alpha^2\, \frac{\partial^2 k_\alpha}{\partial \bar{s}_\alpha^2} \tag{4}$$

In summary, we assume the following evolution equations (of the associative type):

$$\bar{\boldsymbol{l}}_\mathrm{p} = \dot{\boldsymbol{F}}_\mathrm{p} \cdot \boldsymbol{f}_\mathrm{p} = \sum_\alpha \dot{\lambda}_\alpha \frac{\partial \Phi_\alpha}{\partial \bar{\boldsymbol{M}}^t} = \sum_\alpha \dot{\lambda}_\alpha \left[\bar{\boldsymbol{s}}_\alpha \otimes \bar{\boldsymbol{m}}_\alpha\right] \tag{5}$$

$$\dot{k}_\alpha = \dot{\lambda}_\alpha \frac{\partial \Phi}{\partial \kappa_\alpha} = -\dot{\lambda}_\alpha \tag{6}$$

where, in a viscoplastic format, $\dot{\lambda}_\alpha$ can be expressed as

$$\dot{\lambda}_\alpha = \frac{1}{t_*}\left[\frac{\langle \Phi_\alpha \rangle}{C_0}\right]^m$$

A rate-independent solution is obtained if $t_* \to 0$. In the above expression the yield function Φ_α is defined as:

$$\Phi_\alpha = \frac{\bar{\boldsymbol{M}}^t : \left[\bar{\boldsymbol{s}}_\alpha \otimes \bar{\boldsymbol{m}}_\alpha\right]}{b_\alpha} - \kappa_\alpha - \sigma_{\mathrm{y},\alpha} \tag{7}$$

where $\bar{\boldsymbol{M}}^t$ is the elastic Mandel stress and b_α is the integrity on the crystal slip system obtained from $\boldsymbol{b}_\mathrm{d}$.

3 NUMERICAL EXAMPLES

In the numerical examples we have assumed the following boundary conditions: at interior grain boundaries: $\dot{k}_\alpha = 0$ ($\leadsto k_\alpha = 0$), while we assume that $\kappa_{\alpha,\Gamma_0} = 0$ at the external boundaries of the body. The latter condition means that either $\partial k_\alpha / \partial \bar{s}_\alpha = 0$ or $\boldsymbol{N} \cdot \bar{\boldsymbol{s}}_\alpha = 0$. Furthermore, the results shown below are for the case of rate-independence (i.e., $t_* \to 0$). Henceforth, we consider a square Representative Volume Element (RVE).

For simplicity, we assume plane strain for the RVE and that all the grains have only one slip system (the same direction for all the grains in the RVE). This direction is assumed to be 10^o against the horizontal axis. The lower boundary of the RVE is assumed to be fixed in both the vertical and horizontal directions while the upper boundary is subjected to the displacement u in the horizontal direction.

The macroscopic nominal stress F/A vs the macroscopic strain u/L is shown in figure 1 for two values of L.

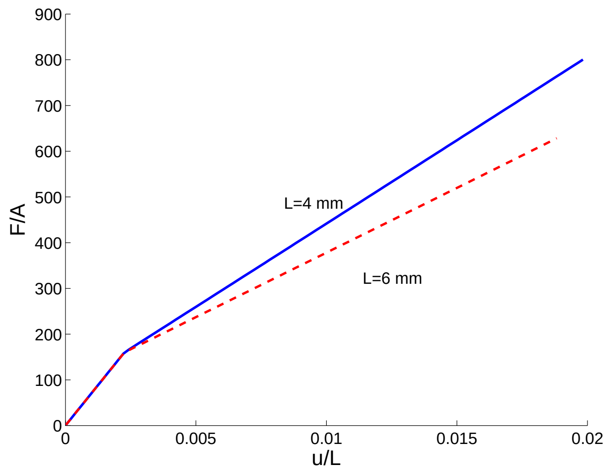

Figure 1: Size dependence of hardening

Figure 2 shows how the plastic slip is distributed at $u/L = 0.02$ for $L = 6$ mm.

4 CONCLUSIONS

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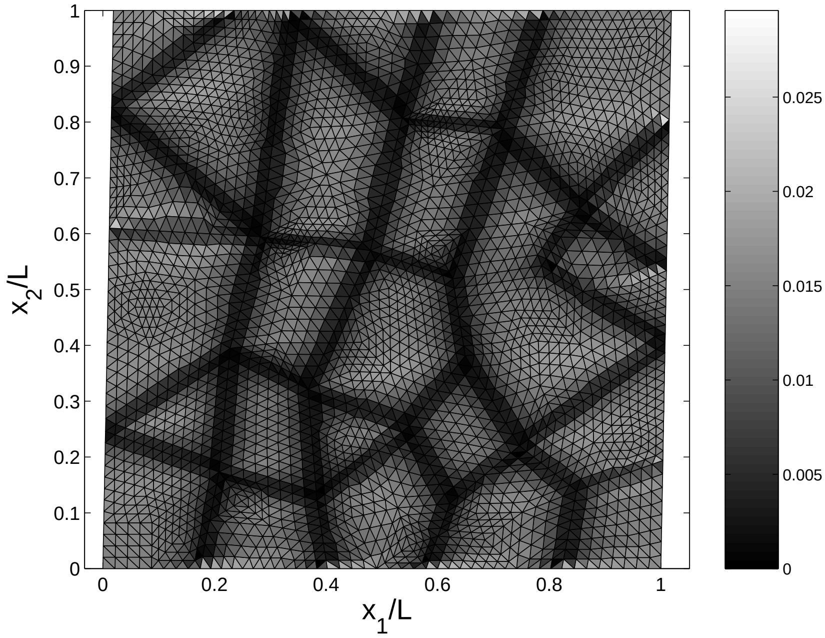

Figure 2: Distribution of plastic strain λ_α for $L = 6$ mm

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