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MODELATION OF DILATANCY EFFECT IN MIXED MODE CRACK OF QUASI-BRITTLE MATERIALS

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1 INTRODUCTION

Some fracture tests[1,2] on quasi-brittle materials like concrete have shown that a non-trivial relation between normal and shear stress in crack faces appears under displacement control with a fix rate between crack opening and crack sliding. This behavior is related to a phenomenon named dilatancy. Here, dilatancy is studied taking into account its influence not only in the “flow rule”[3], but also in the definition of work wasted away by the cracking process[4-6]. The results obtained from them reflect how three considered alternatives of the definition of dilatancy effect influence on model behavior.

2 COHESIVE CRACK MODEL FOR MIXED MODE FRACTURE

A fictitious cohesive crack model for tension-shear (mixed mode I+II) fracture is implemented here within the discrete crack approach, where the discontinuity is modeled in the context of the finite element method through interface elements[3-6]. The fracture behavior is simulated through the cohesive crack stress-separation relation, and the fracture criterion F, which plays the same role as the yield or loading surface in plasticity. F is expressed as[3-6]:

$$F(\underline{S}, \underline{p}) = \tau^2 + \tan^2\phi \; (\sigma - \chi)(2a - \sigma + \chi) = 0 \tag{1}$$

where σ and τ are the normal and shear stresses, $tan\phi$ represents the angle of friction between the crack faces, and χ and a are functions of energy dissipation (W^{cr}) during crack propagation. In the case of Mode I, χ is the cohesive (tensile) stress. Fig. 1 represents Eqn. 1 in a graphical form. For the uncracked material, the constitutive law of the interface element is linear elastic with the stiffness coefficients D_n and D_t.

Once crack starts the hyperbola shrinks because of the dependency of χ and a on W^{cr}, which can be decomposed into the energies dissipated in mode I and mode II,

$$dW^{cr} = dW_I^{cr} + dW_{II}^{cr} \tag{2}$$

In this formulation, the differential of the mode I component of W^{cr} is defined as,

$$dW_I^{cr} = \sigma\, d\omega_n^{cr} \quad if \quad \sigma \geq 0; \qquad dW_I^{cr} = 0 \quad if \quad \sigma < 0 \tag{3}$$

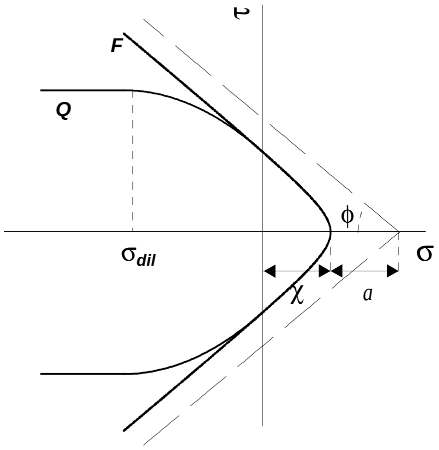

Figure 1: Fracture criterion of the model

and the mode II component in tension as,

$$dW_{II}^{cr} = \tau\, d\omega_t^{cr} \qquad if \quad \sigma \geq 0 \tag{4}$$

In compression three alternatives will be analyzed for W^{cr}_{II}. In one side, classic expression[3,4], in which W^{cr} in shear-compression is the difference between the total energy dissipated in shear and pure friction work. In the other two alternatives dilatancy work is taken into account. The first proposal was initially presented in García-Álvarez[4], which consists on subtracting also dilatancy energy (Eqn. 6), whereas in the second one (López[6]) the expression of W^{cr}_{II} is given by Eqn. 7. Therefore, mode II component of W^{cr} for each proposal is:

$$dW_{II}^{cr} = (|\tau| - |\sigma| \tan\phi)\, |d\omega_t^{cr}| \quad if \;\; \sigma < 0 \tag{5}$$

$$dW_{II}^{cr} = (|\tau| - |\sigma| \tan\phi)\, |d\omega_t^{cr}| \; - \; |\sigma|\, d\omega_n^{cr} \quad if \;\; \sigma < 0 \tag{6}$$

$$dW_{II}^{cr} = \left(|\tau\, d\omega_t^{cr}| - |\sigma|\, d\omega_n^{cr} \right)\left(1 - \frac{|\sigma| \tan\phi}{|\tau|} \right) \quad if \;\; \sigma < 0 \tag{7}$$

In the second proposal dW^{cr}_{II} must be greater or equal to 0, and if in the analysis dW^{cr}_{II} becomes negative, $dW^{cr}_{II}=0$ is imposed. In the third proposal it is possible to demonstrate that dW^{cr}_{II} is always greater or equal to zero.

The components of crack separation can be expressed as the sums of the elastic displacement and real crack opening, where the real crack opening is defined by the flow rule as:

$$d\omega_n^{cr} = d\lambda \frac{\partial Q}{\partial \sigma}; \qquad d\omega_t^{cr} = d\lambda \frac{\partial Q}{\partial \tau} \tag{8}$$

$d\lambda$ is a multiplier, and Q (Fig. 1) is equal to F in tension. In compression, Q is defined as:

$$\frac{\partial Q}{\partial \sigma} = 2\tan^2\phi\ (a + \chi - \sigma)\ f^{dil}\ ; \qquad \frac{\partial Q}{\partial \tau} = 2\,\tau \tag{9}$$

with

$$f^{dil} = \left(1 - \frac{e^{\alpha_d}\ {|\sigma|}/{\sigma_{dil}}}{1 + \left(e^{\alpha_d} - 1\right)\ {|\sigma|}/{\sigma_{dil}}}\right)\frac{a + \chi}{a_0 + \chi_0} \tag{10}$$

Therefore, the crack opening (or dilatancy) is zero when $|\sigma|$ is equal to the parameter σ_{dil} defined in Fig. 1 or when $a + \chi = 0$. α_d parameter lets change the shape of f^{dil} decay. In the uncracked material, $\chi = \chi_0$ (tensile strength) and $a = a_0$ (a parameter related to the shear strength).

3 EXAMPLES AND DISCUSSION

In this section the results obtained from a constitutive analysis will be shown. Two kinds of tests have been considered in order to analyze the influence of considering or not dilatancy energy inside work dissipated during cracking process: 1) shear-compression tests with constant confinement; and, 2) shear tests which are simulation of Hassanzadeh's tests[1], where crack opening and crack sliding are linearly related by a constant (tanγ).

In these two kinds of tests, which are run under displacement control imposing a confinement in the first case, or imposing a fix rate between crack opening and crack sliding in the second one, it may be observed that in a crack opening situation, compression stresses appear in crack faces. This is because crack opening or dilatancy is coerced. The work developed by compression stresses when crack opens is which we call dilatancy energy.

Results of the numerical tests in shear-compression for different confinements are shown in fig. 2: Opening-sliding curves at the left; and, shear-sliding curves at the right. The parameters used are: $D_n = D_t = 10^2$ MPa/mm, $tan\phi = 0.7$, χ_0 =2.8 MPa, a_0 =7 MPa, $G_F^I = 0.1$ N/mm, $G_F^{II} = 1$ N/mm, $\sigma_{dil} = 30$ MPa, y $\alpha_d = 2$. In this figure it is observed that: 1) when confinement level increase, the influence of the dilatancy decrease; and, 2) if dilatancy energy is considered in the analysis, the area under shear-sliding (ω_n^{cr}) increases.

In fig. 3 numerical results obtained are compared with experimental results obtained by Hassanzadeh for γ equal to 30º. This modelation was made using the parameters cited above, except a_0, G_F^{II}, $tan\phi$ (values in the figure), and $\alpha_d = 0$. The results shown here are preliminary results (only 30º case was fitted), but it may be yet observed, from a qualitative point of view, that: 1) Comparing the right graph of fig. 2 with the graphs of fig. 3 we can note that in this second kind of tests, it is where to take into account dilatancy energy inside the energy balance of the model plays an important role; and, 2) comparing (Fig. 3) numerical fit obtained for γ=30º with results from γ=60º, it can be concluded that dilatancy effect increase when shear (mode II) component increase.

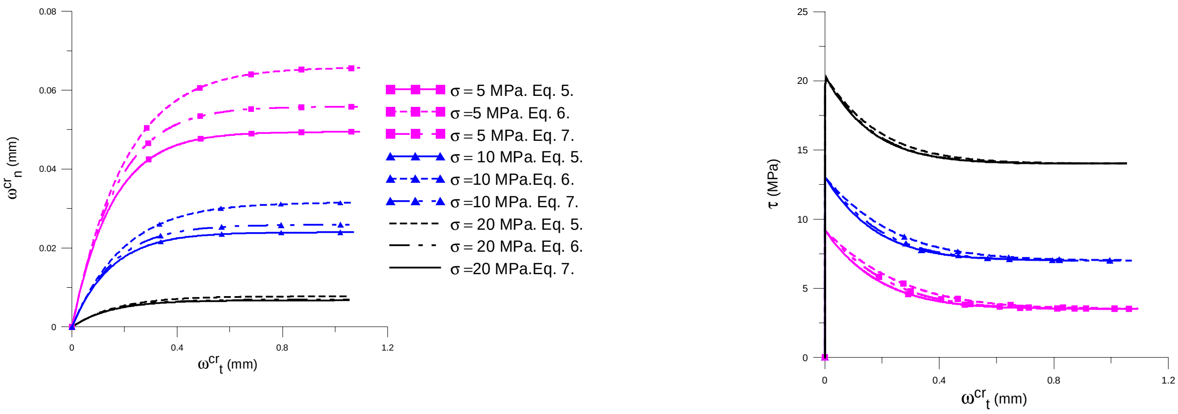

Figure 2: Results obtained in shear-compression with constant confinement examples

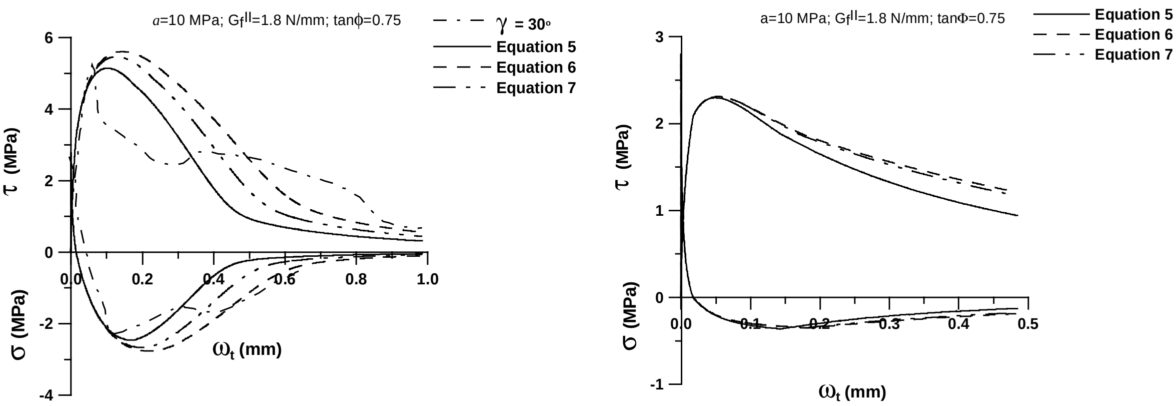

Figure 3: Comparison of experimental and numerical results of Hassanzadeh's tests for γ equal to 30º. Incidence of dilatancy energy for γ equal to 60º

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