VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D. R. J. Owen (Eds)

A MOISTURE-DEPENDENT DAMAGE-PLASTICITY MODEL FOR MODE I LOADING

Peter Moonen[*], Staf Roels[*] and Jan Carmeliet[*†]

[*] Laboratory of Building Physics, Department of Civil Engineering, Katholieke Universiteit Leuven,
Kasteelpark Arenberg 40, 3001 Leuven, Belgium.
Web page: http://www.kuleuven.be/bwf/eng/

[†] Building Physics Group, Faculty of Building and Architecture, Technical University Eindhoven, P.O.
box 513, 5600 MB Eindhoven, The Netherlands.
Web page: http://sts.bwk.tue.nl/fago/

Key words: Damage-Plasticity, Coupled model, hygro-mechanical loading.

1 INTRODUCTION

The presence of water significantly influences the mechanical response of porous materials due to the activation of molecular forces and surface energy phenomena[1,2]. Among other effects, moisture causes a decrease in stiffness and a reduction of the tensile strength of the material. Loading cycles are characterized by stiffness degradation and the occurrence of permanent deformations after unloading. All these phenomena can be adequately described by a combined damage-plasticity model.

In the present paper the constitutive equation of a combined continuum damage-plasticity model[3,4] are adapted to be consistent with embedded strong discontinuities and to account for the influence of moisture. The model is implemented in a fully coupled discrete crack model[5]. The model is illustrated by means of a numerical example of a 3 point bending test.

2 KINEMATICS OF STRONG DISCONTINUITIES

The displacement field of a body crossed by m non-intersecting discontinuities is given by[6]

$$u = \hat{u} + \sum_{i=1}^{m} H_{\Gamma_i} \tilde{u}_i \tag{1}$$

where $\hat{u}$ and $\tilde{u}_i$ are smooth, continuous functions on Ω and H_{Γ_i} is the Heaviside step function corresponding to the i[th] discontinuity ($H_{\Gamma_i} = 1$ if $x \in \Omega^+$ and $H_{\Gamma_i} = 0$ if $x \in \Omega^-$).

The strain field of a body crossed by a discontinuity can be found by taking the symmetric gradient of the displacement field:

$$\boldsymbol{\varepsilon} = \nabla^s \mathbf{u} = \nabla^s \hat{\mathbf{u}} + \sum_{i=1}^{m} H_{\Gamma_i} \nabla^s \tilde{\mathbf{u}}_i + \sum_{i=1}^{m} \delta_{\Gamma_i} \left(\tilde{\mathbf{u}}_i \otimes \mathbf{n}_i \right)^s \tag{2}$$

where $\mathbf{n}_i$ is the normal to the i[th] discontinuity and δ_{Γ_i} is the Dirac delta distribution.

3 DISCRETE DAMAGE-PLASTICITY MODEL

The combined damage-plasticity formulated by Simo & Ju [3,4] uses the following constitutive equation

$$\boldsymbol{\sigma} = (1-d)\mathbf{C}^e\left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p\right) \text{ with } d = 1 - \frac{q\left(\kappa^d\right)}{\kappa^d} \tag{3}$$

where $\boldsymbol{\sigma}$ is the stress tensor, d is the damage variable, κ^d is a history parameter, $\mathbf{C}^e$ is the elastic constitutive tensor, $\boldsymbol{\varepsilon}$ is the total strain tensor and $\boldsymbol{\varepsilon}^p$ is the plastic strain tensor. The model should be consistent with an incorporated discontinuity. Inserting the kinematical description of the strain field (Eq. 2) in the constitutive equation (Eq. 3) yields

$$\boldsymbol{\sigma} = (1-d)\mathbf{C}^e\left(\nabla^s\hat{\mathbf{u}} + \sum_{i=1}^{m} H_{\Gamma_i}\nabla^s\tilde{\mathbf{u}}_i + \sum_{i=1}^{m}\delta_{\Gamma_i}\left(\tilde{\mathbf{u}}_i \otimes \mathbf{n}_i\right)^s - \boldsymbol{\varepsilon}^p\right) \tag{4}$$

Assuming that the plastic strain field can be decomposed in a similar form as the total strain field and allowing only plasticity at the discontinuity, yields

$$\boldsymbol{\sigma} = (1-d)\mathbf{C}^e\left(\nabla^s\hat{\mathbf{u}} + \sum_{i=1}^{m} H_{\Gamma_i}\nabla^s\tilde{\mathbf{u}}_i + \sum_{i=1}^{m}\delta_{\Gamma_i}\left(\left(\tilde{\mathbf{u}}_i \otimes \mathbf{n}_i\right)^s - \left(\tilde{\mathbf{u}}_i^p \otimes \mathbf{n}_i\right)^s\right)\right) \tag{5}$$

The stress field must remain bounded and therefore the Dirac-delta distribution must cancel. Following Oliver[7], we do this by making the internal damage variable unbounded.

$$\kappa^d = \bar{\bar{\kappa}}^d + \sum_{i=1}^{m}\delta_{\Gamma_i}\bar{\kappa}_i^d \tag{6}$$

Substituting Eq. 6 into Eq. 3, assuming an elastic behavior in the continuum ($\bar{\bar{\kappa}}^d = 0$) and taking into account that the value of δ_{Γ_i} is zero everywhere, except at the ith discontinuity, the stress field in a point on discontinuity Γ_i becomes

$$\boldsymbol{\sigma}_{\Gamma_i} = \frac{q\left(\bar{\kappa}_i^d\right)}{\bar{\kappa}_i^d}\mathbf{C}^e\left(\left(\tilde{\mathbf{u}}_i \otimes \mathbf{n}_i\right)^s - \left(\tilde{\mathbf{u}}_i^p \otimes \mathbf{n}_i\right)^s\right) \tag{7}$$

Multiplying Eq. 7 by the normal to the discontinuity and rearranging gives an expression for the traction forces

$$\mathbf{t}_i = \boldsymbol{\sigma}_{\Gamma_i}\mathbf{n}_i = (1-\omega_i)\mathbf{Q}_i^e\left(\tilde{\mathbf{u}}_i - \tilde{\mathbf{u}}_i^p\right) \text{ with } \omega_i = 1 - \frac{q\left(\bar{\kappa}_i^d\right)}{\bar{\kappa}_i^d} \text{ and } -\infty \le \omega_i \le 1 \tag{8}$$

with ω_i the degenerated damage variable and $\mathbf{Q}_i^e$ the elastic acoustic tensor, both related to the ith discontinuity. Eq. 8 shows that the total separation at the ith discontinuity is split into a recoverable damage part and an irrecoverable plastic part.

4 IMPLEMENTATION AND NUMERICAL EXAMPLE

4.1 Crack initiation and propagation

A crack is initiates or propagates if the plasticity yield surface, expressed in the principal stress space, is violated. For the case of mode I loading, a Rankine failure surface is often adopted (Eq. 9)

$$f^{p} = \sigma_I - f_t(S) \leq 0 \tag{9}$$

Where σ_I is the first principal stress and f_t is the tensile strength of the material. Based on the findings of Carmeliet and Van den Abeele[8], the dependency of f_t on the degree of saturation of the material can be expressed as

$$f_t(S) = E(S)\,\varepsilon_{cr} \text{ where } E(S) = E_{S=0} + \left(E_{S=1} - E_{S=0}\right)\cdot S^{1/2} \tag{10}$$

with $E_{S=0}$ the Young's modulus at dry state, $E_{S=1}$ the Young's modulus at vacuum saturation state and ε_{cr} the critical strain level in the material. For the Rankine surface expressed in Eq. 9, the discontinuity will grow perpendicular to the maximal principal stress direction. Crack path continuity is imposed. For more complex models, the direction of crack growth should be based on the non-local stress field at the crack tip[9] .

4.2 Behavior at the discontinuity

After initiation of the discontinuity, the cohesive zone behavior is stated in terms of tractions and separations. The magnitude of the traction forces (Eq. 8) is obtained in two steps. First, the plasticity problem is solved in the effective stress space, then the damage variable is updated and the effective stresses are mapped back to the homogenized stress space.

If plastic deformations are assumed to occur in the undamaged material bonds, the plastic yield function can be expressed in terms of tractions in the effective stress space[10]

$$f^{p} = \hat{T}_n - h\kappa^{p} \leq 0 \tag{11}$$

Where $\hat{T}_n$ is the effective normal traction force, h is the hardening modulus of the material and κ^p is the internal plastic variable. In a first step the separation increment is considered fully elastic. Trial tractions are calculated and used to evaluate the plastic yield surface. If $f^p < 0$ the yield function is not violated, and the trial tractions correspond to the effective tractions. When this is not the case, the trail tractions must be corrected such that the final stress state is situated on the yield surface $f^p = 0$. This condition determines the value for the plastic multiplier.

The damage variable can be updated (Eq. 8) and the stresses can be mapped onto the actual stress space. The model is illustrated for the case of a 3 point bending test subjected to loading, unloading and reloading (Fig. 1). The effect of moisture is clearly visible.

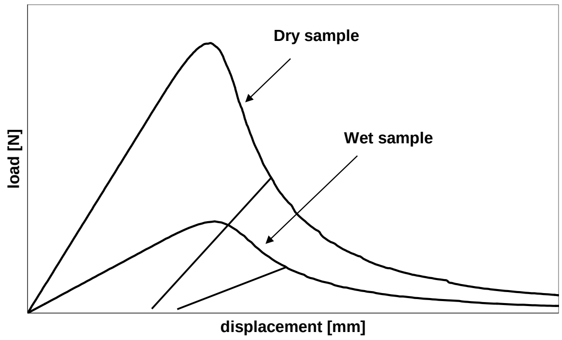

Figure 1: Load displacement diagram for a dry and a fully saturated beam subjected to a 3-point bending test

5 CONCLUSIONS

A combined damage plasticity model for mode I loading has been derived from a continuum model. The model has been adapted to take into account effects of moisture on the mechanical behaviour and has been implemented into a fully coupled discrete crack model. The derivations can be easily extended to more complex models.

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