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ELASTOPLASTICITY WITHIN THE FRAMEWORK OF MICROPLANE MODELS FOR GEOTECHNICAL APPLICATIONS

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Summary. *Microplane theory has proved to be a strong numerical tool for the advanced analysis of engineering materials. However, the application of this theory within the constitutive framework of elastoplasticity needed to represent the real behaviour of geotechnical materials is a complex problem, due to the nature of this type of models. In this paper, the coupling of elastoplasticity and microplane theory for geological media will be presented. A theoretical discussion will be followed by a description of applicable models for their use in geotechnical analysis.*

1 INTRODUCTION

During the past few years it has been proved that microplane theory, combined with other constitutive techniques such as elastoplasticity, has a great potential in the field of advanced material modelling, especially in the case of Geomechanics. However, real engineering applications of this type of models have not yet reached the mainstream. The main reason is that while the fundamental theoretical framework has been completed for cohesive-frictional quasi-brittle materials[1–8], there are a considerable amount of material parameters that have no clear physical meaning and have to be adjusted or fixed in advance. In Geomechanics, the materials have been formed in a long and complex process, and it would be almost impossible trying to identify the very large amount of parameters needed to represent the materials behaviour. Moreover, geological materials range from hard rocks, of quasi-brittle behaviour, to soft rocks and soils which exhibit a more plastic behaviour. A general model for this wide collection of materials would have no practical engineering applications unless its behaviour can be modelled with only a short number of easily-measurable parameters. One way to achieve this is to formulate the microplane elastoplastic constitutive law based on parameters that can be related to a classical invariant-based macroscopic model. This relation must be straightforward, and therefore requires a precise knowledge of the relations between the microplane parameters and the macroscopic response of the model.

A geotechnical engineering applicable constitutive model should be expressed in terms of parameters that can be determined from common field or laboratory tests. The model should be capable of reproducing the basic aspects of geotechnical behaviour such as strain-hardening/softening, pressure sensitivity, dilatancy, dependence of stiffness on the stresses and strains, etc. For more advanced applications the model should also be able to reproduce anisotropy. And last but not least, the model must be based on a yield or failure surface with an adequate shape according to geotechnical behaviour.

2 PRINCIPLES OF MICROPLANE ELASTOPLASTICITY

The traditional microplane models have been usually developed with a strain-driven formulation in which the strain vector acting on a plane of arbitrary orientation (the "microplane") is equal to the projection of the strain tensor on the same plane [2,3,9,10]. A thermodynamically consistent stress-strain relation, derived from a potential or free energy was developed by Carol et al [4].

A consistent formulation of an elastoplastic microplane model within a classic framework of small-strain elastoplasticity, for which plastic deformation is the main dissipative mechanism, is available in the literature [4,5]. However, the work carried out is mainly formal in terms of thermodynamics consistency and applications do not go beyond simple cases.

In the early approach to microplane elastoplasticity [7] the model developed considered volumetric-deviatoric split of the normal components (necessary in that formulation to achieve Poisson's ratio greater than 0.25) as an important feature and gave satisfactory results in the curve fitting process. However elastoplasticity was only considered in the shear components and it was impossible to establish an acceptable relationship between the adjustable parameters of the explicit volumetric and deviatoric laws and the cohesive frictional constants of the shear law. Even more difficult was to find their macroscopic, measurable counterparts. In the last decade, efforts have been made to solve these difficulties by several authors [3,5,6,8]. See a full discussion in Sánchez & Prat (2005) [11].

It can be seen [11], however, that defining the model with the volumetric-deviatoric split induces a lack of an elasticity modulus for the (whole) normal components, which is necessary to calculate the normal stress increments: the volumetric-deviatoric split will necessarily cause that, for some orientations, the volumetric strain (or stress) can have different sign than the deviatoric strain (or stress). In those microplanes it happens frequently that the sum of the strain components has a different sign (i.e., direction) than the sum of the stress components, thus having positive normal stresses with negative normal strains or viceversa.

How can this be understood? For any plane (defined by its normal vector **n**), introducing the static and kinematic constraints into Hooke's law, one can obtain the normal component of the stress acting on such a plane and its elastic relation with the normal strain:

$$\sigma_N = \left[2\mu + 3\lambda\left(\varepsilon_V / \varepsilon_N\right)\right]\varepsilon_N = \hat{E}_N \varepsilon_N \tag{1}$$

Eq. (1) shows that $\hat{E}_N$ depends on the state of deformation and the orientation of the plane considered, and does not necessarily have a positive sign, since $\varepsilon_V / \varepsilon_N$ can be negative. Thus, one may conclude that there is no direction-independent or positive-definite relation between normal components of stress and strain in the overall space.

Looking in a more detailed manner into the question of stress and strain projections over prescribed directions in space, one can see that the deviatoric part of the normal component belongs to the same set of continuum quantities than the tangential part. This means that they should not be treated separately. Together, both portions belong to the projection of the deviatoric overall strain or stress tensor on the arbitrarily oriented plane:

$$\left\|\sigma_D^{\text{mic}}\right\| = \left[(\sigma_{D_1}^{\text{mic}})^2 + (\sigma_{D_2}^{\text{mic}})^2 + (\sigma_{D_3}^{\text{mic}})^2\right]^{1/2} = \left[(\sigma_{D_N}^{\text{mic}})^2 + (\tau^{\text{mic}})^2\right]^{1/2} \tag{2}$$

For geotechnical applications, it seems convenient to re-define the kinematic constraint such that the microplane deviatoric strain vector is complemented with the volumetric portion of strain, acting with equal magnitude on every microplane, just as in traditional microplane models. Defining the consistent microplane stresses as work conjugates of the strains [4,12], the overall stress tensor can be split into its volumetric and deviatoric portions and relate each part with the integral of the microplane stress conjugates to the macroscopic strains:

$$\sigma_{ij} = p\delta_{ij} + s_{ij}; \qquad p\delta_{ij} = \frac{3}{2\pi}\int_\Omega \sigma_V \frac{\delta_{ij}}{3} d\Omega; \qquad s_{ij} = \frac{3}{2\pi}\int_\Omega \sigma_{D_r}\left[\frac{1}{2}\left(\delta_{ri}n_j + \delta_{rj}n_i\right) - \frac{1}{3}\delta_{ij}n_r\right] d\Omega \tag{3}$$

Following analogous procedures as in traditional microplane models in order to define the elastic constants one gets the moduli $E_V = 3K$ and $E_D = E_T = 2G$, where K and G are the macroscopic bulk and shear modulus respectively.

3 BASIC MODELS

For geotechnical engineering applications, only models that exhibit dependency on Lode's angle θ and on the mean pressure p are of real interest [13]. Because of space constraint only the more common Mohr-Coulomb-type microplane model will be summarized here. This type of yield criterion is defined at the microplane Mohr space as:

$$F^{\text{mic}} = \tau^{\text{mic}} - c^{\text{mic}} - \sigma_N^{\text{mic}} \tan\varphi^{\text{mic}} \le 0 \tag{4}$$

This uniquely defined yield function implies the existence of different yield surfaces at the microplane volumetric–generalized deviatoric space for every differently oriented microplane (Figure 1b). The yield function in this space is:

$$F^{\text{mic}} = \left\|\sigma_D\right\| - k^{\text{mic}} - \sigma_V^{\text{mic}} \tan\alpha^{\text{mic}} \le 0 \tag{5}$$

where k^{mic} and α^{mic} depend on the microplane strength parameters and the stress path [13]. Eq. (5) is similar to the classical Mohr-Coulomb criterion.

$$F = \tau - \left[c/\tan\varphi - p\right] g(\theta) \le 0 \tag{6}$$

Fig. 1 shows the strong dependency of the microplanes' response to the discretisation model selected. It can be seen that the 64-plane model generates at the macroscopic deviatoric π-plane a smooth yield locus which can be much approximated to complex geotechnical failure criteria such as Lade's model [14], while the 28-plane model simply drifts apart too much from realistic values while working close to a Lode's angle of 30º.

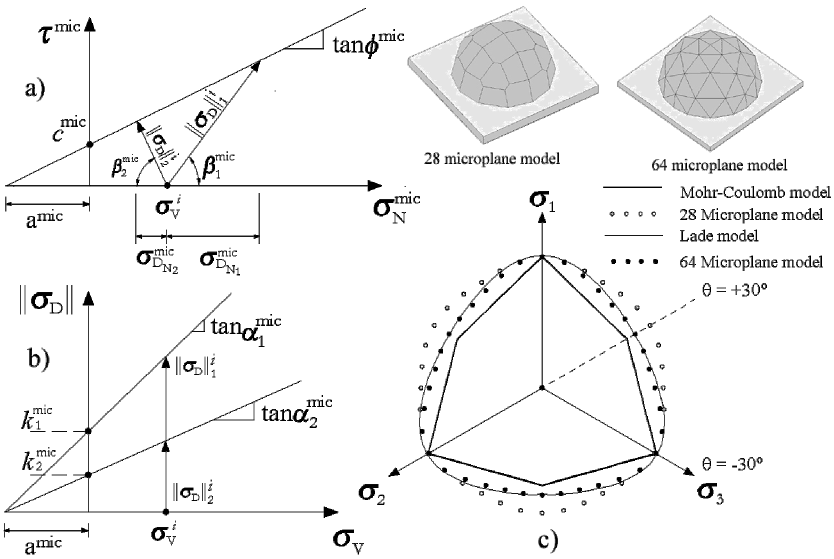

Figure 1: a) Microplane yield surface at the $\sigma_N^{mic} - \tau^{mic}$ stress space; b) various yield surfaces at the space $\sigma_V^{mic} - \|\sigma_D\|$; c) Macroscopic yield loci at the π-plane for 28 and 64 microplane models and the classical Mohr-Coulomb and Lade models

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