VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D. R. J. Owen (Eds)
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MODELING OF INELASTIC RESPONSE OF THICK RC SHELL WITH ENHANCED SOLID ELEMENTS

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Key words: DOF Enrichment, Reinforced Concrete, Strong Discontinuities, XFEM.

Summary. *In this work, we present a new Enhanced Solid Element for reinforced concrete, which is able to overcome not only the different nonlinear behavior of each component (concrete, steel & bonding), but also each particular kinematics. Inspired of XFEM techniques (enrichment functions and level set method), the Enhanced Solid Element is able to take into account the sliding between steel and concrete, responsible for bonding deterioration. Concerning material behaviors, they are based on a thermodynamics formulation: concrete damage is characterized by an alternative model based on "strong discontinuity approach", bonding is represented by an elasto-plastic-damage constitutive model, and steel by a classical elasto-plastic constitutive model.*

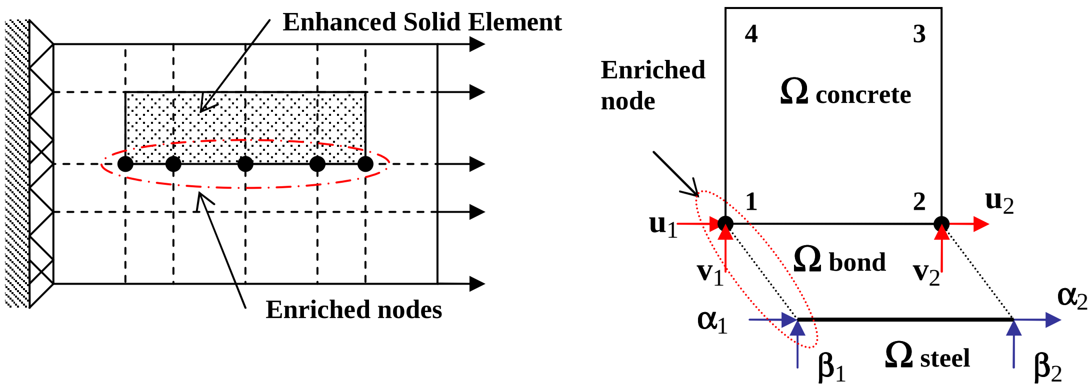
1 INTRODUCTION

Use of Reinforced Concrete as the main structural component in Civil Engineering constructions is a very common practice since its invention in the nineteenth century. For this complex hybrid material, a crucial piece which governs the apparition and development of cracks in concrete. Total modeling of this kind of structures has always been very complex and so, in spite of bonding importance, this one is often supposed to remain perfect. Nevertheless, recent design requirements impose to provide not only the guaranties of structural integrity but also structural durability related to given crack patterns, which imposes a fresh start in modeling of reinforced concrete.

Some finite elements based on homogenized material behavior have already been formulated[6] to improve RC analysis. Moreover, a recent model taking into account bonding deterioration by implementing a local stiffness reduction[2] has been developed. However, these models do not include any considerations about kinematics of each particular material. Our objective is to develop a new Enhanced Solid Element for numerical analysis of massive RC structures. This new finite element will be able to: a) take into account each particular kinematics concrete, steel & bonding; b) analyze internally the specific nonlinear behavior of each material, without homogenization; c) preserve continuity of steel rebars between different finite elements; and d) simplify mesh complexity and numerical resolution of the problem.

2 DESCRIPTION OF THE ENHANCED SOLID ELEMENT

Inspired of XFEM techniques[3], the key point of the Enhanced Solid Element is the “Enrichment of Degrees of Freedom” in some of its nodes. Basically, we ‘enhance’ the nodes where the steel rebar should be placed explicitly (see Fig. 1(b)). By using the “Level Set Method” [5], we can find the geometrical position of the rebar inside of each element, and choose the nodes to be enriched. In a different way of classical XFEM, Shape Functions and their derivatives are defined separately for each material domain, and integrated uniquely over the concerned integration point. An elementary stiffness matrix will be built with, and assembled to the structure global matrix in the standard manner. Even if all equations can be calculated simultaneously, a sequential resolution based on matrix partition might be implemented, in order to comply with convergence requirements.



(a) (b)

Figure 1: (a) discretization with Enhanced Solid Elements; (b) enriched nodes and element internal composition

In order to support this formulation, we have adopted the next hypotheses:

- All external forces are applied over the concrete domain.
- Bonding and steel volume forces are neglected in formulation, because of their weak influence on structure’s global response.
- Each material’s nonlinear behavior will be described in the thermodynamics framework.
- Steel rebar is always placed on one of the edge of the Enhanced Solid Element.

3 THE ENHANCED SOLID ELEMENT FORMULATION

3.1 DOF enrichment and shape functions

In our case, the enriched approximation for the element’s nodal displacement field is written as follows:

$$\mathbf{u}^h(x) = \sum_{i=1}^{4} N_i^{\mathrm{C}}(x)\cdot \mathbf{u}_i + \left[\sum_{i=1}^{2} N_i^{\mathrm{B}}(x)\cdot \mathbf{u}_i + \sum_{i=1}^{2} N_i^{\mathrm{B}}(x)\cdot \boldsymbol{\alpha}_i\right] + \sum_{i=1}^{2} N_i^{\mathrm{S}}(x)\cdot \boldsymbol{\alpha}_i \tag{1}$$

In this expression, N_i^m are the isoparametric shape functions in each domain, $\mathbf{u}_i$ are displacements corresponding to the standard DOF, and $\boldsymbol{\alpha}_i$ are the displacements corresponding to the enriched DOF activated only in concerned nodes. Concerning shape functions, each group is calculated from the geometrical characteristics of each domain. For bonding shape functions, a geometric parameter h_{pen}, successfully used in non-width interface elements[4], is introduced in order to avoid Jacobian singularities.

3.2 Discrete formulation and numerical integration

After linearization of the weak form of the equilibrium equation, we obtain the following equation to solve:

$$\mathop{\mathrm{A}}_{e=1}^{N_{elem}} \left[\mathbf{K}_{\mathrm{C}}\Big|_{PG=1-4} \cdot \mathbf{u}^{(i)} + \mathbf{K}_{\mathrm{B}}\Big|_{PG=5,6} \cdot \begin{Bmatrix} \mathbf{u}^{(i)} \\ \boldsymbol{\alpha}^{(i)} \end{Bmatrix} + \mathbf{K}_{\mathrm{S}}\Big|_{PG=7} \cdot \boldsymbol{\alpha}^{(i)} = \mathbf{f}^{ext} - \mathbf{r}^{e(i)} \right] \tag{3}$$

Being $\mathop{\mathrm{A}}_{e=1}^{N_{elem}}$ the element assembly operator, $\mathbf{K}_m$ are the stiffness matrix of each domain $m(Concrete, Bonding, Steel)$, $\mathbf{f}^{ext}$ is the external force vector, and $\mathbf{r}^{e(i)}$ correspond to residual. Numerical integration is performed by using the following expressions:

$$\mathrm{k}_{ij}^{C} = \int_{\Omega^B} \left(\mathrm{B}_i^C\right)^{\mathrm{T}} \mathrm{C}_C^{inelastic}\, \mathrm{B}_j^C \,\mathrm{d}\,\Omega_C \qquad (i, j = \mathrm{u}, \mathrm{v}),\ \mathrm{k}_{ij}^{S} = \int_{\Omega^A} \left(\mathrm{B}_i^S\right)^{\mathrm{T}} \mathrm{C}_S^{inelastic}\, \mathrm{B}_j^S \,\mathrm{d}\,\Omega_S \qquad (i, j = \alpha, \beta) \tag{4}$$

$$\mathrm{k}_{ij}^{B} = \int_{\Omega^L} \left(\mathrm{B}_i^B\right)^{\mathrm{T}} \mathrm{C}_B^{inelastic}\, \mathrm{B}_j^B \,\mathrm{d}\,\Omega_B \qquad (i, j = \mathrm{u}, \mathrm{v}, \alpha, \beta)$$

4 MATERIAL CONSTITUTIVE MODELS

The nonlinear behavior of each component is described by different constitutive models, formulated in a thermodynamics framework. Expressed in stress-strain terms, the behavior is calculated specifically in each integration point (Gauss quadrature).

4.1 Concrete (Strong discontinuity formulation)

Modeling of nonlinear behavior of concrete is very important for global response of the structure. In order to avoid localization problems, we have chosen a strong discontinuity formulation which is able to take account of damage in concrete[1].

4.2 Steel

We have adopted a classical elasto-plastic constitutive model for steel.

4.3 Bonding

An elastic-plastic-damage model for bonding (recently developed and tested by the authors[4]), was adopted in the formulation due to its various advantages: a) thermodynamics formulation, written in stress-strain terms; b) capacity of coupling between: “cracking and frictional sliding”, and “tangential and normal stresses”; c) able to take account of confinement influence and lateral pressure; d) great stability for monotonic and cyclic loading.

5 NUMERICAL EXAMPLES

Some numerical examples, comparing results from different kinds of discretization for a same problem (standard without bonding, standard with interface elements and using Enhanced Solid Elements) will be presented on the Conference.

6 CONCLUSIONS

- In this work, we have presented a new brand formulation for analysis of Reinforced Concrete structures, the Enhanced Solid Element, which is able to take account particular kinematics as well as nonlinear behavior of each of the components: concrete, bonding and steel. Based on some XFEM techniques, we propose a nodal DOF enrichement, as well as the construction of an elementary stiffness matrix by assembling the three different domains.
- For material constitutive models, we have adopted a strong discontinuity formulation for concrete damage, an elasto-plastic-damage model for bonding, and a standard elasto-plastic behavior for steel.
- A method of sequential resolution is proposed in order to reduce complexity of calculations as well as a better convergence.

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