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USE OF THE ELASTOPLASTIC 1D BEAM ELEMENT TO EVALUATE THE LIMIT BEHAVIOUR OF 2D STEEL FRAMES UNDER ELEVATED TEMPERATURES.

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Summary. *A general procedure is presented to evaluate the limit behaviour in the 2D elastoplastic beam element under static forces and thermal loading. We assume the Navier hypothesis for beams and sudden and concentrated plasticity in the beam-ends attending the yield function Y based on the von Mises criterion. Results illustrate that in a beam with plastic behaviour the fixed-end forces depend not only on the applied forces and temperature distribution but also on the forces and temperatures that caused the plasticity in the considered end of the beam and that coupling between forces and displacements appears in the cross-section in plastic state.*

1 INTRODUCTION

We present an analytical formulation based in the *finite element method* for the nonlinear structural analysis of frames incorporating thermal effects. It is well known that the usual numerical way for the determination of the plastic collapse load of framed structures is the use of one-dimensional finite element models (2D beam element) together with the *plastic hinge* concept and an incremental procedure. The approach we present takes into account both material and geometric nonlinearities from an analytical point of view, using the *extended plastic cross section* concept (that includes the plastic hinge concept) and needing not only incremental but also iterative algorithms (*plastic return*). We try to apply this approach to the analysis of 2D frames under static and thermal loading. The main assumptions made in the modelling include the *Navier* hypothesis for beams, the *von Mises* yield criterion together with associated flow rule for plastic behaviour and linear temperature profiles in any cross section for thermal loading. We also consider that steel loses strength and stiffness for rising temperatures and we use the expressions included in EC3-1.2 (CEC 1995). The load is applied in a quasi-static way using sequential load curves and buckling effects and reversal loading are neglected.

The model represents the steel members by two-noded one-dimensional line elements (beams) with thermal loading (point or distributed inter-element loading is also possible using the same strategy) (figure 1). The plasticity is supposed to be concentrated[1] only at the nodes and once a node is in plastic state the combination of the axial force N, shear force V and bending moment M satisfies the yielding condition[2] Y(N,V,M,T) at any temperature T. The tangent stiffness matrix is required for the non linear solution procedure. In each increment, for increasing temperature or increasing load, it is necessary a return iterative algorithm[3] at any plastic node in order to assure the plastic requirements.

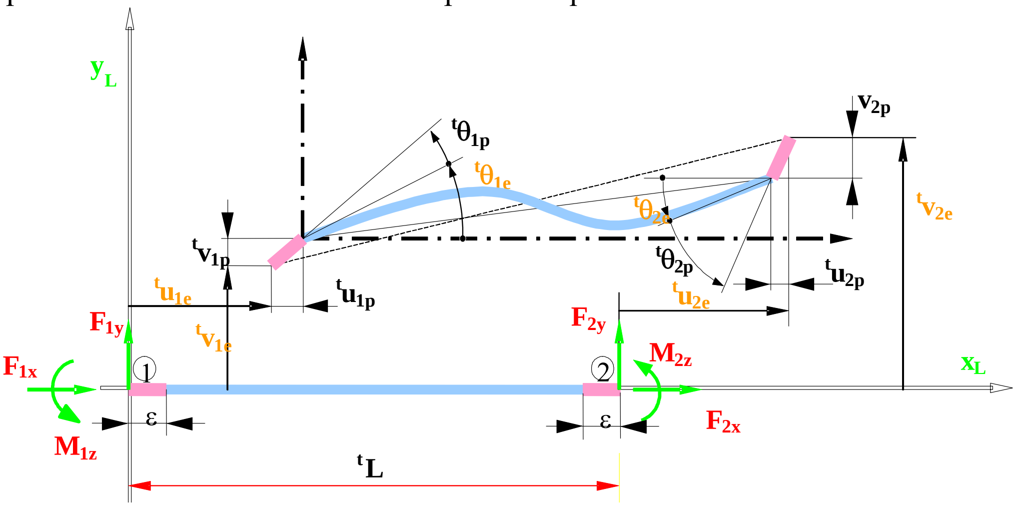
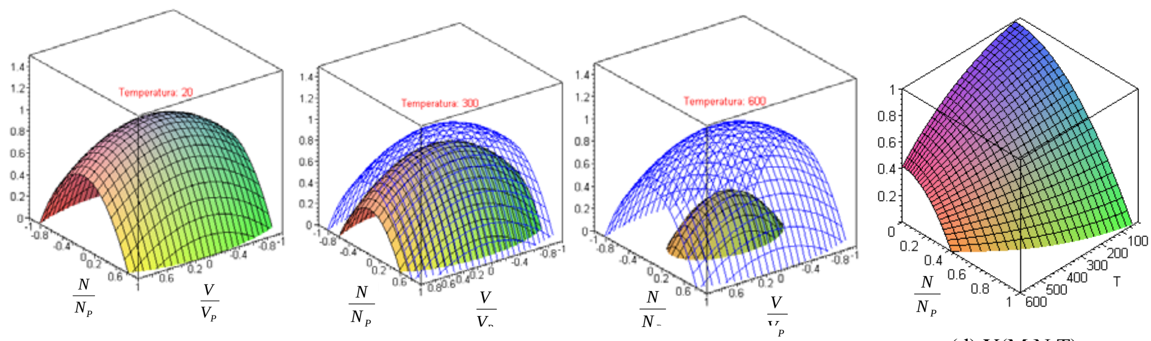


Figure 1: 2D Beam finite element with the possibility of plastic behavior in its ends

2 YIELD FUNCTION

As it is known[1,2,6,7], the yield function Y is the relationship between N, V and M that leads the considered section to be in plastic state for any temperature T. Bearing in mind that some material properties depends on the temperature (we assume the expression of EC3) we present[5,7] the resulting yield function for 3 different temperatures in figure 2,a,b,c and for any temperature in figure 2,d (neglecting shear force V). N_P, V_P and M_P are, as usual, the fully plastic tensile force, shear force and bending moment, respectively. Figure 2,a shows[6,7] the Y function at reference temperature whereas 2,b and 2,c shows furthermore Y at 300 and 600 ºC



(a) 20ºC (b) 300ºC (c) 600ºC (d) Y(M,N,T)

Figure 2. Effects of the temperature on the yield function.

3 FINITE ELEMENT FORMULATION

We assume the standard FEM formulation for non-linear structural analysis based on the tangent stiffness matrix K. Under an *updated lagrangian approach* and bearing in mind that the total or thermo-elastoplastic displacement can be decomposed[2,7] into its thermo-elastic and themo-plastic components, the incremental beam forces are related to incremental displacements and temperatures by the equation

$$
\begin{aligned}
\{dF\} = \{\Delta F^{Tep}\} + \{\Delta F^{Te}\} + \{\Delta F^{Tp}\} = \left[[K] - \frac{[K]\left\{\frac{\partial Y}{\partial F}\right\}\left\{\frac{\partial Y}{\partial F}\right\}^t [K]}{-\left\{\frac{\partial Y}{\partial u^p}\right\}^t \left\{\frac{\partial Y}{\partial F}\right\} + \left\{\frac{\partial Y}{\partial F}\right\}^t [K] \left\{\frac{\partial Y}{\partial F}\right\}} \right] \{du^{Tep}\} + \\
\left[[dK] - \frac{[K]\left\{\frac{\partial Y}{\partial F}\right\}\left\{\frac{\partial Y}{\partial F}\right\}^t [dK]}{-\left\{\frac{\partial Y}{\partial u^p}\right\}^t \left\{\frac{\partial Y}{\partial F}\right\} + \left\{\frac{\partial Y}{\partial F}\right\}^t [K] \left\{\frac{\partial Y}{\partial F}\right\}} \right] \{u^{Te}\} - \left\{ \frac{[K]\left\{\frac{\partial Y}{\partial F}\right\}\left\{\frac{\partial Y}{\partial T}\right\}^t dT}{-\left\{\frac{\partial Y}{\partial u^p}\right\}^t \left\{\frac{\partial Y}{\partial F}\right\} + \left\{\frac{\partial Y}{\partial F}\right\}^t [K] \left\{\frac{\partial Y}{\partial F}\right\}} \right\}
\end{aligned}
\tag{1}
$$

where Y is the yield function and F is the vector of beam forces (N, V and M in both nodes). This equation shows that for any element the beam forces can be obtained by adding 3 parts: the forces due to the thermo-elastoplastic displacements u^{Tep}, the ones due to the thermo-elastic displacements u^{Te} and another part due only to thermo-plasticity. From this governing equation (1) and assembly for several elements in the usual way it is possible to obtain the incremental governing equation for any frame. Nevertheless it is necessary to know the effect of the thermal loading in terms of its equivalent effect at the beam ends.

4 FIXED END FORCES

For the determination of the forces at the beam-ends equivalent to the considered thermal loading we use the same strategy that Ibán[7] for interelemental distributed forces. The beam is divided in 3 elements (figure 3) and ΔT_1 and ΔT_2 is acting only in the elastic central one (so it is easy to obtain its equivalent forces) whereas the other two elements may include the plastic behavior. After solving this virtual frame for L_1 and L_3=0 the solution can be found. It can be shown[7] that the solution depends not only on ΔT_1 and ΔT_2 (and of course on E, A and I_Z) but also, for beams with some plastic node, on the N_0, V_0 and M_0 forces that led that node to plasticity.

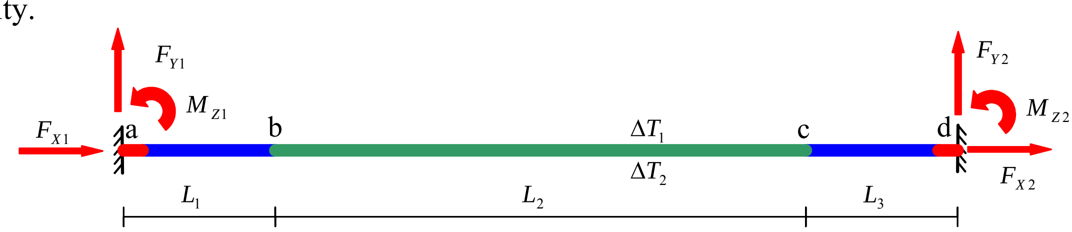


Figure 3. Virtual discretization.

5 EXAMPLE

We suppose that in a thermal loaded frame there is a certain beam in which axial force and bending moment at, for example, its left-end have followed the path shown in figure 4 for a monotonic rising temperature. For simplicity we suppose that yield function depends only on M, N and T. N and M reach, at point 1, the corresponding yield curve for the current temperature T_1 so that plastic behavior begins in that end. After and increment of temperature (T_2) there is a change in the path (point 2, evaluated according (1) using tangent matrix). From this point 2 it is necessary the iterative return algorithm in order to return this point to its corresponding yield curve T_2 (point 2')

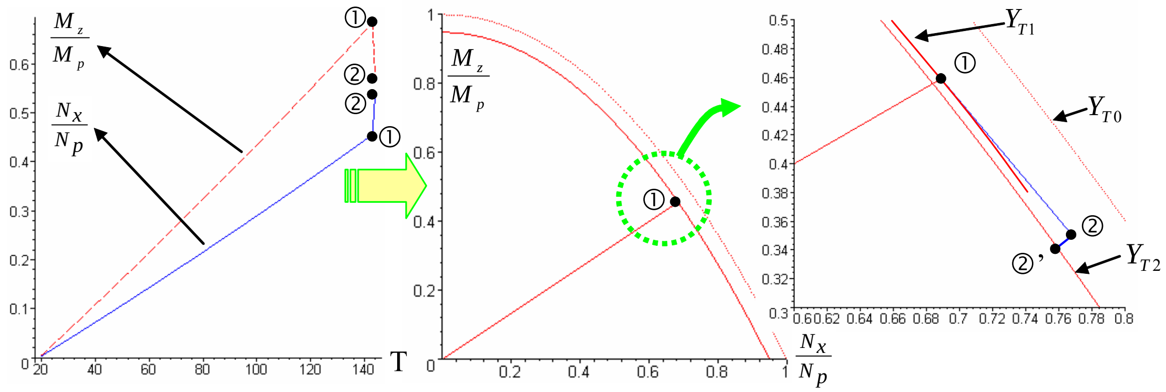


Figure 4. Loading path.

6 CONCLUSIONS

This work presents a numerical formulation of 2D beam finite element with plasticity and with properties depending on the temperature and some numerical results for a particular case. This formulation is similar to the one for the thermoelastoplastic problem in continuum mechanics and hence needs for tangent stiffness matrix, yield function, plastic return, etc. but not expressed on stresses but on beam forces (N, V and M). The resulting method is more rigorous that the classical plastic methods for frames based in plastic hinges and can describe with better accuracy the behavior of frames under forces and thermal loading.

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