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FORMULATIONS FOR LOCKING TREATMENT IN ANISOTROPIC LARGE DEFORMATION ELASTO-PLASTIC THIN STRUCTURES

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Summary. *In the present work the authors show results obtained with the implementation of an Enhanced Assumed Transverse Strain methodology for thin shells, taking into account anisotropic material nonlinearities, in problems involving large deformations, displacements and rotations.*

Transverse shear and membrane locking patterns are removed from the original formulation, solely based on the displacement field. The resultant bilinear (4 nodes) shell finite element, fully integrated and with 20 nodal degrees-of-freedom, does not rely on any other mixed formulation. The enhanced strain field is properly designed to enlarge the null transverse shear strain subspace coming from the classical degenerated formulation. At the same time, a minimum number of enhanced variables is involved in the core formulation. A shell element can then be attained with the same predictability as one based on a reduced integrated formulation, but not incorporating spurious energetic modes.

Non-linear effects are treated in a local reference frame, only affected by the rigid-body part of the total deformation. Additive and multiplicative update procedures for the finite rotation degrees-of-freedom are implemented, in order to correctly reproduce mid-point configurations along the incremental deformation path. Stress and strain tensors are additively updated in the local frame, together with the enhancing strain field. The final enhanced strain tensor come from a straightforward implementation of nonlinear geometric and material relations. The accuracy of the implemented formulation can be shown in a set of isotropic and anisotropic elasto-plastic problems, in smooth and non-smooth shell structures. Obtained results with the present formulation are in close agreement with either simulation and experimental data.

1 STATE OF THE ART AND MAIN GOALS OF THE WORK

Finite shell elements technology started with the development of the so-called "degeneration approach", introduced by Ahmad *et al.*[1], where a class of shell elements were obtained from the continuum, resorting to the concept of a "reference" surface. Plane-stress constitutive conditions have automatically been included into the formulation, in a way to make elements numerically consistent with the modelling of thin structures. Since then, isoparametric shell elements incorporating displacement and rotation-like degrees-of-freedom have been continuously improved, with extensions to different kinds of problems being presented in a wide set of fundamental works. A "complete" list of relevant contributions would be extensive, however, and beyond the scope of this paper. A comprehensive study on the evolution of shell elements technology can be reported to the prominent work of Yang *et al.*[2].

Sooner, however, it was verified that this class of elements, particularly the bilinear ones (treated in the present work), was prone to show locking effects in the thin shell limit. For this case, transverse shear strain energy did not vanish at all points within the element domain for pure bending deformations, contradicting the Kirchhoff-Koiter-Love hypothesis for membrane structures. The situation is even worse as it is clearly difficult to establish a definite boundary between situations where a given formulation tends to "lock" or not, with locking appearance being not just a matter of thickness values (or ratio between thickness and a dominant dimension)[3].

In order to circumvent transverse shear locking phenomena, alternative techniques such as the selective reduced integration, mixed interpolation of tensorial components and, finally, the assumed natural strain have proved to be successful approaches. The common point with these formulations is the fact that the original, displacement-based, strain field is judiciously replaced by substitute components which are interpolated in distinct (non-conventional) ways.

The Enhanced Assumed Strain (EAS) method, from the classical set of papers by Simo, Rifai, Armero and Taylor[4–6], departs from the previous works, keeping the original strain field unchanged, then adding additional strain terms, chosen in conformity to the desired improvement in a given element's formulation. In this way, the same formalism can be applied to a set of pathologies appearing in lower-order finite element formulations, such as the in-plane locking (2D elements), the volumetric locking (3D elements) or membrane locking, for plates and shells[7;8].

This was the starting point for the work of the authors. Departing from the conventional degenerated approach applied to bilinear (four-node) fully-integrated shell elements, a complete analysis of the null transverse shear strain subspace was performed in[9]. Along with the present work, this reference points to previous developments by the authors in EAS-based two-dimension[10;11], shell[12;13] and solid-shell[14;15] finite elements technology.

For the specific case of shell elements[13], the likelihood of transverse shear locking to happen was related to the ability (or not) of a given formulation to successfully reproduce the correct solution by naturally providing enough components to the subspace basis[9]. The main advantage of this subspace analysis was that its validity remains unaltered irrespective to the loading and boundary conditions applied to a given element, being a characteristic of a given formulation. Comparison of the subspace bases provided by the degenerated approach, the selective reduced integration method and the mixed interpolation of tensorial components have revealed missing components in the first formulation. Instead of performing a replacement of transverse shear strain terms to correct the element formulation, the EAS method was then used to additively improve the transverse shear strain field coming from the displacement-based formulation. This have led to a set of shell elements with distinct number of internal variables and with no transverse shear locking for a variety of linear test cases[9]. A preliminary study on large rotation problems, while keeping elastic strains, was performed by the authors in[12] with encouraging results.

In the present work, the previous shell formulation is revised and improved, from both the fundamental and computational aspects. Comparatively, the approach here presented allows for the use of a lower number of EAS internal variables, with gains of computational time and efficiency. This study accomplishes for results coming from both smooth and non-smooth shell structures, while accounting for nonlinear geometric and material models. Related to the latter topic, isotropic as well as planar and normal anisotropic yield criteria are implemented.

Results obtained with the shell formulation, along with the adopted constitutive laws, are compared to those provided by experimental and simulation data from well-established formulations in the literature. The detailed formulation, as well as implementation aspects, can be found in the most recent references from the authors[13;16].

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