

# EXPERIMENTS AND NUMERICAL ANALYSIS OF ANISOTROPICALLY DAMAGED ELASTIC-PLASTIC SOLIDS

Michael Brüning\* and Marcílio Alves†

\* Lehrstuhl für Baumechanik-Statik, University of Dortmund,  
D-44221 Dortmund, Germany  
E-mail: michael.bruenig@uni-dortmund.de

† Department of Mechatronics and Mechanical Systems Engineering,  
University of São Paulo, São Paulo SP 05508-900, Brazil  
E-mail : maralves@usp.br

**Key words:** Anisotropic damage, ductile metals, experiments, finite element analyses.

**Summary.** *The paper deals with the continuum modelling of the large strain elastic-plastic deformation behavior of anisotropically damaged ductile metals. Numerical simulation of the deformation process of tension specimens will demonstrate the applicability of the proposed damage theory. Tension tests on aluminum alloys are performed to be able to identify material parameters and to verify the continuum approach.*

## 1 INTRODUCTION

The accurate and realistic description of inelastic behavior of ductile materials as well as the development of associated efficient and stable numerical solution techniques is essential for the solution of boundary-value problems occurring in various engineering disciplines. Large inelastic deformations of metals are usually accompanied by damage processes due to microdefects. Their proper understanding and their mechanical description are of importance in discussing the mechanical effects of the material deterioration on the macroscopic behavior of solids as well as in elucidating the mechanisms leading to final fracture. From a practical point of view, continuum models intended to represent anisotropic damage phenomena should be simple enough to allow efficient numerical treatment and identification of material parameters but at the same time its simplicity should not eliminate the essential features of the deformation behavior within the range of application. Therefore, based on the concepts of continuum damage mechanics, an efficient constitutive model is proposed and the identification of material parameters and its experimental verification are discussed.

## 2 EXPERIMENTS

Static tensile tests were performed in three aluminum alloys whose bulk of material consists of sheets with thickness of 6.35mm, 3.175mm and 1.6mm. Each alloy was tested in

the main lamination direction and perpendicular to it in order to access the degree of anisotropy. The specimens were cut in a dog bone shape with the minimum cross-section having dimensions of the sheet thickness by 10mm. Electronic extensometers were fixed in the specimens in order to monitor the deformation. This was used together with the reading of a calibrated load cell to obtain the change in the elastic modulus as the plastic deformation increases along the various load-unload cycles (see Alves et al.<sup>1</sup> for further details). A typical result of the measured curve is shown in Figure 1 for the sheet 3.175mm thick.

### 3 ANISOTROPIC CONTINUUM DAMAGE MODEL

The framework presented by Brünig<sup>2</sup> is used to describe the inelastic deformations of ductile materials including the evolution of anisotropic damage due to microdefects. Briefly, the effective specific free energy  $\bar{\phi}$  of the undamaged matrix material is decomposed into an effective elastic and an effective plastic part

$$\bar{\phi} = \bar{\phi}^{el}(\bar{\mathbf{A}}^{el}) + \bar{\phi}^{pl}(\gamma) \quad (1)$$

where  $\bar{\mathbf{A}}^{el}$  is the effective elastic strain tensor and  $\gamma$  denotes an internal plastic variable. This leads in the case of isotropic elastic material behavior to the effective stress tensor

$$\bar{\mathbf{T}} = 2G \bar{\mathbf{A}}^{el} + \left( K - \frac{2}{3}G \right) \text{tr} \bar{\mathbf{A}}^{el} \mathbf{1} \quad (2)$$

where  $G$  and  $K$  represent the shear and bulk modulus of the undamaged material, respectively. In addition, plastic yielding is assumed to be adequately described by the yield condition

$$f^{pl}(\bar{I}_1, \bar{J}_2, c) = \sqrt{\bar{J}_2} - c \left( 1 - \frac{a}{c} \bar{I}_1 \right) = 0, \quad (3)$$

where  $\bar{I}_1 = \text{tr} \bar{\mathbf{T}}$  and  $\bar{J}_2 = \frac{1}{2} \text{dev} \bar{\mathbf{T}} \cdot \text{dev} \bar{\mathbf{T}}$  are invariants of the effective stress tensor  $\bar{\mathbf{T}}$ ,  $c$  denotes the strength coefficient of the matrix material and  $a$  represents the hydrostatic stress coefficient.

Moreover, the Helmholtz free energy function of the damaged material sample is assumed to consist of three parts:

$$\phi = \phi^{el}(\mathbf{A}^{el}, \mathbf{A}^{da}) + \phi^{pl}(\gamma) + \phi^{da}(\mu). \quad (4)$$

The elastic free energy  $\phi^{el}$ , which is an isotropic function of the elastic and damage strain tensors,  $\mathbf{A}^{el}$  and  $\mathbf{A}^{da}$ , is used to describe the elastic response of the damaged material. The energies  $\phi^{pl}$ , due to plastic hardening, and  $\phi^{da}$ , due to damage strengthening, only take into account the respective internal state variables,  $\gamma$  and  $\mu$ . This yields the Kirchhoff stress tensor of the damaged material sample

$$\mathbf{T} = 2\left(G + \eta_2 \text{tr} \mathbf{A}^{da}\right) \mathbf{A}^{el} + \left[ \left( K - \frac{2}{3} G + 2\eta_1 \text{tr} \mathbf{A}^{da} \right) \text{tr} \mathbf{A}^{el} + \eta_3 \left( \mathbf{A}^{da} \cdot \mathbf{A}^{el} \right) \right] \mathbf{1} + \eta_3 \text{tr} \mathbf{A}^{el} \mathbf{A}^{da} + \eta_4 \left( \mathbf{A}^{el} \mathbf{A}^{da} + \mathbf{A}^{da} \mathbf{A}^{el} \right) \quad (5)$$

which is linear in  $\mathbf{A}^{el}$  and  $\mathbf{A}^{da}$ , and  $\eta_1 \dots \eta_4$  are newly introduced material parameters taking into account the deterioration of the elastic material properties due to damage. In addition, onset and continuation of anisotropic damage is assumed to be governed by the damage criterion

$$f^{da}(I_1, J_2, \tilde{\sigma}) = I_1 + \tilde{\beta} \sqrt{J_2} - \tilde{\sigma} = 0 \quad (6)$$

where  $\tilde{\beta}$  represents the influence of the deviatoric stress state on the damage condition and  $\tilde{\sigma}$  denotes the equivalent aggregate stress measure.

Elastic and plastic material parameters are identified from the tension tests discussed above. In particular, the elastic constants are chosen to be  $G = 27000$  MPa and  $K = 58000$  MPa. To be able to characterize the effective plastic behavior, the nonlinear increase of the current strength coefficient  $c$  appearing in the yield condition (3) is numerically simulated by the power law

$$c = c_0 \left( \frac{H_0 \gamma}{n c_0} + 1 \right)^n \quad (7)$$

The initial yield strength  $c_0 = 186$  MPa, the initial hardening parameter  $H_0 = 7300$  MPa, and the hardening exponent  $n = 0.157$  give the best fit to experimental values. In addition, progressive damage often results in strain softening of the aggregate and the corresponding stress-strain curve exhibits a negative slope. The unloading paths of our tests give information on the onset and quantity of damage so allowing the identification of further material parameters. The four additional material parameters  $\eta_1 \dots \eta_4$  appearing in the elastic constitutive equation (5), however, which describe the deteriorating influence of increasing damage on the elastic properties of the aggregate, are difficult to be determined. Therefore, Brünig and Ricci<sup>3</sup> studied numerically the effect of these parameters on the deformation and localization behavior of tension specimens. They showed that a variation of the damage parameters  $\eta_1 \dots \eta_4$  does not remarkably affect load-deflection curves. Therefore, it was suggested that these parameters should be exemplarily determined using experimental data on ductile metals given by Spitzig et al.<sup>4</sup> and the parameters are:

$$\eta_1 = -117500 \text{ MPa}, \eta_2 = -95000 \text{ MPa}, \eta_3 = -190000 \text{ MPa} \text{ and } \eta_4 = -255000 \text{ MPa}.$$

#### 4 NUMERICAL SIMULATION

The deformation behavior of the aluminum tension specimen tested above is numerically analyzed. Figure 1 shows the experimental load – engineering strain curve including the un- and reloading paths. In addition, the numerically predicted curve based on plane strain finite

elements is also plotted and it shows good agreement with the experimental data. Therefore, the presented continuum damage model is able to realistically simulate the deformation behavior of tension specimens up to final fracture.

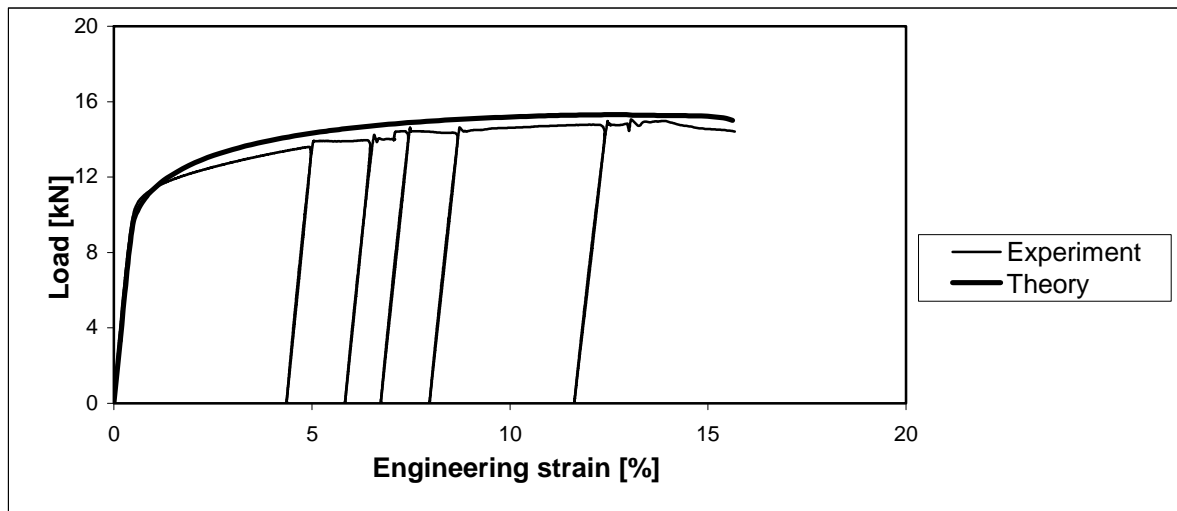


Figure 1: Load –engineering strain curves

## 5 CONCLUSION

A numerical model for the analysis of ductile elastic-plastic-damage metals has been presented. Experiments with aluminum alloys have been used to identify material parameters and to verify the approach. Hence, the present anisotropic continuum damage model may be used to solve practical engineering problems including service life prediction of metal structures.

## REFERENCES

- [1] M. Alves, J. Yu, N. Jones, “On the elastic modulus degradation in continuum damage mechanics”, *Comput. Struct.* 76, 703-712 (2000).
- [2] M. Brünig, “An anisotropic ductile damage model based on irreversible thermodynamics”, *Int. J. Plasticity* 19, 1679-1713 (2003).
- [3] M. Brünig, S. Ricci, “Nonlocal continuum theory of anisotropically damaged metals”, *Int. J. Plasticity* 21, 1346-1382 (2005).
- [4] W.A. Spitzig, R.E. Smelser, O. Richmond, “The evolution of damage and fracture in iron compacts with various initial porosities”, *Acta Metall.* 36, 1201-1211 (1988).