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COMPARISON OF FUZZY SET THEORY AND STOCHASTIC METHOD RESULTS IN APPLICATION TO THE ANALYSIS OF THE ULTIMATE LOAD-CARRYING CAPACITY OF STEEL MEMBERS WITH IMPERFECTIONS

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Summary. *The analysis of the load-carrying capacity of a steel hot-rolled strut under axial compression is presented in the article. The load-carrying capacity is solved with respect to the second order theory. The Fuzzy analysis results are compared with results obtained from the statistical analysis, evaluated using the Monte Carlo numerical simulation. Acquired results of the load-carrying capacity of the axially compressed strut are also compared with results obtained according to standards EN1990 and EUROCODE 3.*

1 INTRODUCTION

The contribution will show the use of this method to model indeterminacies. Indeterminateness can, for our purposes, be divided into two subgroups: vagueness and randomness. The theory of fuzzy sets is used in modelling vagueness, whilst the probability theory is used when modelling randomness [1, 3]. Prof. Lotfi Zadeh used the notion "fuzzy" for the first time in 1962. In 1965, L. Zadeh published the paper, legendary at present, "Fuzzy sets"[6]. The fuzzy theory answers the question of what happened, whilst the probability theory answers the question of whether or not an event occurred. The probability gives us information of the frequency of the occurrence of an event, while the fuzzy set theory limits the given event.

The fuzzy analysis of the load-carrying capacity of a strut under compression solved using the second order theory is shown in the paper. The fuzzy analysis of the load-carrying capacity determinates the concept, without taking into account the occurrence or non-occurrence of the event. The theory of fuzzy sets measures the degree to which an event occurs, not whether it occurs. The load-carrying capacity is solved analytically. The fuzzy analysis results are compared with the results obtained from the stochastic analytical solution using the Monte Carlo (MC) simulation for 10000 runs. The load-carrying capacity is also calculated deterministically according to standards EUROCODE 3 [8] and EN1990 [9].

2 GENERAL SPECIFICATIONS

Fuzzy numbers, defined on a set of real numbers, will be used in the article. As a rule it is assumed, that they have a special shape, as shown on fig. 1. The Fuzzy number represents a value, which is inaccurate, and can be characterized verbally as „maybe a_0“, (see Fig. 1). In the Fuzzy set theory, basic arithmetic with fuzzy numbers is developed (addition, subtraction, multiplication, division). The description of the practical applications of these operations in specific situations for all fields is listed for example in [3, 5]. The result of the operation is a fuzzy number, to which a membership function appertains (see Fig. 1).

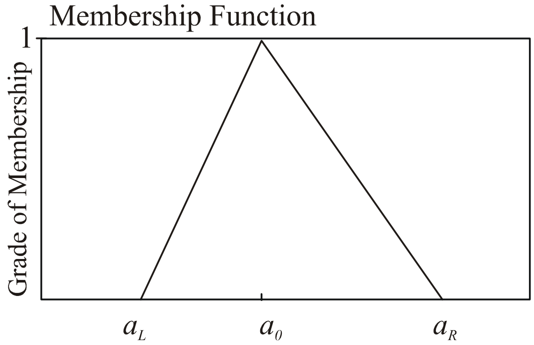

Figure 1: Fuuzy number

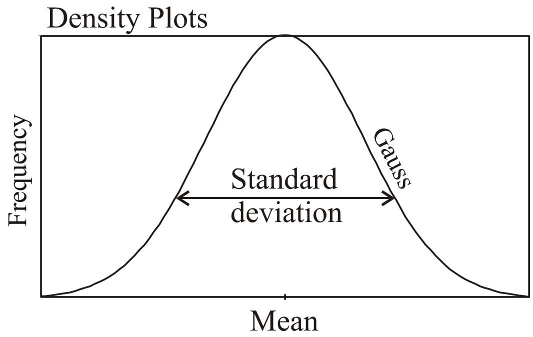

Figure 2: Random variable

The Membership function has nothing in common with probability. In probability we investigate the frequency of the occurrence of a phenomenon, which is precisely defined. The frequency of the occurrence of an investigated phenomenon is characterized using a probability density function (see Fig. 2). The aim of the article is the analysis of both types of indeterminacies.

3 LOAD-CARRYING CAPACITY

The load-carrying capacity of a strut under compression was identified as the compressive force, under which the plasticization of the strut in the most stressed section starts during the exceeding of the yield strength f_y. Buckling perpendicular to the web was modeled. Profile HEB 300U with initial imperfections in the form of the sine function was considered. The load-carrying capacity N of the strut can be determined acc. to the relation:

$$\delta_x = \frac{N}{A} + \frac{N \cdot e_0}{W \cdot \left(1 - N / N_{cr}\right)} = f_y \qquad \Rightarrow N \tag{1}$$

Where A is the cross-sectional area, N is axial strut force, W is the section modulus, e_0 is the amplitude of the initial sinusoidal curvature of the strut axis, N_{cr} is Euler’s critical force, which is for a bilaterally hinged strut defined as $N_{cr} = \pi^2 EI/L_{cr}^2$. A bilaterally hinged strut of length L = 7.2665m and non-dimensional slenderness ratio acc. to [8], $\overline{\lambda} = 1.0$ was solved.

4 INPUT QUANTITIES AS FUZZY AND RANDOM NUMBERS

First input variable is yield strength. We can assume, for the yield strength, that the membership function is formative identical with the density function, which was obtained by the evaluation of experimental results [2]. Due to small skewness and kurtosis, the distribution can be assumed as Gaussian, see Fig. 3.

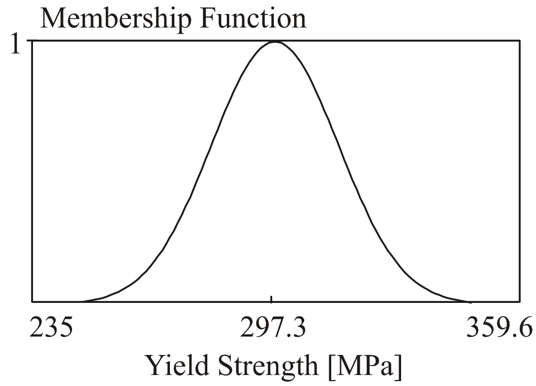

Figure 3: Fuzzy number: Yield strength

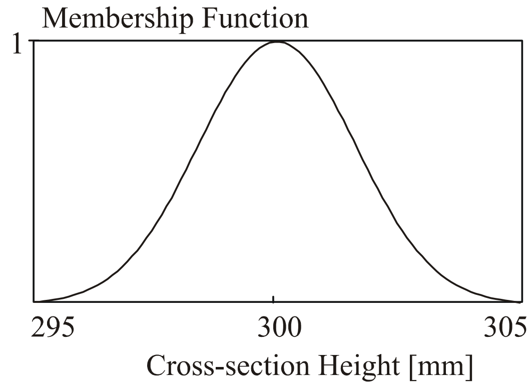

Figure 4: Fuzzy number: Cross-section height

Further we will assume, that the membership function of geometric characteristics of profile HEB 300 *h, b,* t_1, t_2 (see Tab.1) are formative identical with the Gaussian distribution acc. to Tab. 1. In the event, that the height of profile HEB 300 is equal to the nominal value of 300 mm, the degree of truth of the statement, that it is the height of profile HEB 300 is 100%, i.e. equals 1, see Fig. 4. The membership functions of the other variables *b*, t_1, t_2, e_0, *E*, are defined in a similar manner, using statistical characteristics from Tab. 1. (see Tab.1.).

In the second alternative, input variables are considered as random variables, see Tab. 1. Statistical characteristics of the geometric characteristics were identified from the presumption, that in tolerance limits of standards [6] 95 % of realization are found. Statistical distribution of the amplitude of the initial sinusoidal curvature e_0 was approximately determined assuming, that 95 % of realization rest within the interval $\langle 0; L/1000 \rangle$. The modulus of elasticity *E* was introduced acc. to [4].

No.	Quantity	Name of random quantity	Type of distribution	Dimens ions	Mean value	Standard deviation
1.	f_y	Yield strength	Gauss	MPa	297.3	16.8
2.	h	Cross-section height	Gauss	mm	300	1.5
3.	b	Flange width	Gauss	mm	300	2
4.	t_1	Web thickness	Gauss	mm	11	0.75
5.	t_2	Flange thickness	Gauss	mm	19	1
6.	e_0	Amplitude of curvature	Gauss	mm	4	2
7.	E	Young's modulus	Gauss	GPa	210	10

Table 1 : Input random quantities

5 CONCLUSIONS

The comparison of histogram obtained by MC simulation with the membership function of random load-carrying capacity obtained acc. to (1) is shown on Fig. 5. The membership function (full line) was obtained by the aid of so-called α-cuts [3] for ten layers. The membership function has indispensable non-zero value, even where the frequency of occurrence of random variables is virtually zero. For comparison deterministic design values obtained acc. to standards [8], [9] as 0.1 percentile are listed.

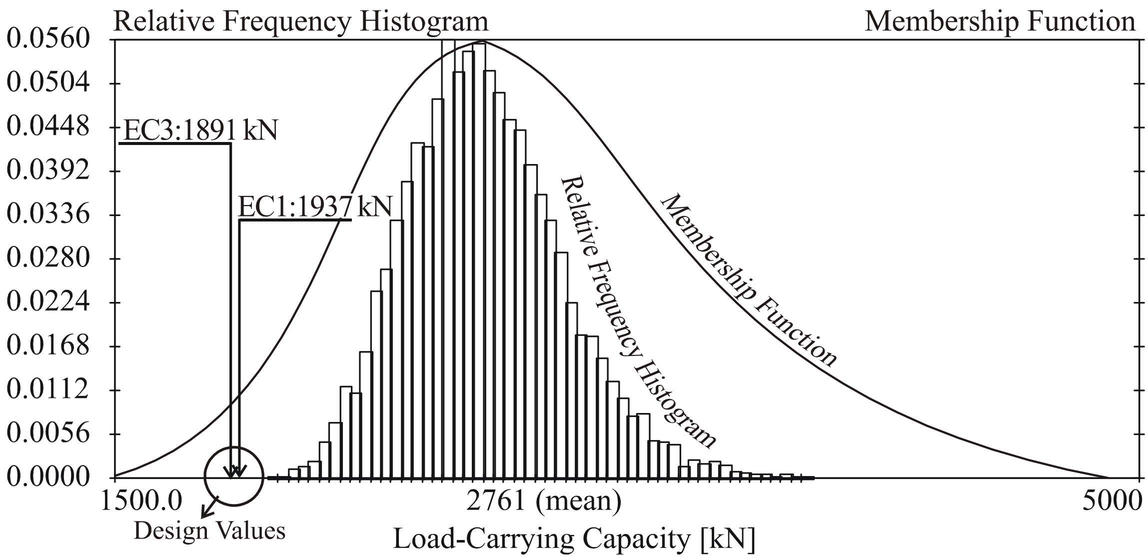

Figure 5: Comparison of the stochastic and fuzzy analysis

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