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MULTISCALE MODELING OF THE THERMOVISCOPLASTIC BEHAVIOR OF METAL MATRIX COMPOSITES

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Summary. *This paper deals with the thermoviscoplastic behavior of metal matrix composites. A constitutive model, based on Dvorak's Transformation Field Analysis, is implemented to investigate the response of an $Al/Al_2 0_3$ composite under an anisothermal, cyclic loading. The experimental validation of the model and its implementation in the finite element code ABAQUS through a user defined material (UMAT) subroutine are discussed.*

1 INTRODUCTION

Metal matrix composites (MMCs) are promising lightweight materials with high strength-to-weight and rigidity-to-weight ratios coupled with attractive thermal insulation properties. Short fiber MMCs are being considered in the automotive industry as potential candidates for local reinforcements of engine components subjected to severe thermomechanical loadings.

It is the purpose of this paper to present a constitutive model for such materials. The model relies on a multiscale approach, and is based on Dvorak's Transformation Field Analysis. The constituents' constitutive equations along with the derived effective constitutive model are presented in the second section. Time integration of the model is described in the third section, while the experimental validation is discussed in the fourth section.

2 MICRO-MECHANICAL MODELING

The metal matrix composite under study consists of an Al matrix and Al_2O_3 short fibers. At operating temperatures, the fibers can be considered as linearly elastic, the elasticity tensor of which is denoted by $\mathbf{L}^{(f)}$, and the metal as viscoplastic, the constitutive response of which is described using a unified model of cyclic viscoplasticity based on the nonlinear kinematic hardening rule[1]:

$$\boldsymbol{\sigma} = \mathbf{L}^{(\mathsf{m})}(T)\,\boldsymbol{\epsilon}_{\mathsf{e}} = \mathbf{L}^{(\mathsf{m})}(T)\,(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\mathsf{vp}} - \boldsymbol{\epsilon}_{\mathsf{th}}) \qquad \boldsymbol{\epsilon}_{\mathsf{th}} = \boldsymbol{k}(T)(T - T_{\mathsf{ref}}), \tag{1}$$

$$\dot{\boldsymbol{\epsilon}}_{\mathsf{vp}} = \frac{3}{2}\left\langle \frac{J_2(\boldsymbol{\sigma}-\boldsymbol{x}) - \sigma_y(T)}{\eta(T)} \right\rangle^{n(T)} \frac{\mathsf{dev}(\boldsymbol{\sigma}-\boldsymbol{x})}{J_2(\boldsymbol{\sigma}-\boldsymbol{x})} \tag{2}$$

$$\boldsymbol{x} = \frac{2}{3}H(T)\,\boldsymbol{\alpha} \qquad \dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\epsilon}}_{\mathsf{vp}} - \frac{3}{2}\,\dot{p}\,\xi(T)\,\boldsymbol{\alpha} \tag{3}$$

where $\langle\rangle$ is the Macauley bracket, $\boldsymbol{\sigma}_{\mathsf{y}}$, n, η, H et ξ are temperature-dependent material parameters with their usual meanings, subscripts e, vp and th refer to elastic, viscoplastic and thermal, respectively, $\mathbf{L}^{(\mathsf{m})}$ is the matrix elasticity tensor, T_{ref} is the reference temperature for zero stress, p is the cumulated plastic strain, $\boldsymbol{k}$ is the tensor of thermal expansion, dev is the deviator operator, and $J_2(\boldsymbol{\sigma}-\boldsymbol{x}) = \sqrt{\frac{3}{2}\mathsf{dev}(\boldsymbol{\sigma}-\boldsymbol{x})\cdot\mathsf{dev}(\boldsymbol{\sigma}-\boldsymbol{x})}$.

The effective elastic properties of the MMC are estimated using Ferrari's model[2]:

$$\mathbf{L}^{\mathsf{eff}} = \mathbf{L}^{(\mathsf{m})} + c^{(\mathsf{f})}\,\left\{\left(\mathbf{L}^{(\mathsf{f})} - \mathbf{L}^{(\mathsf{m})}\right)\,\mathbf{T}^{(\mathsf{f})}\left(c^{(\mathsf{m})}\mathbf{I} + c^{(\mathsf{f})}\mathbf{T}^{(\mathsf{f})}\right)^{-1}\right\} \tag{4}$$

where $\{.\}$ is the average on all orientations of fibers, f and m stand for the fibers and the matrix respectively, c is the volume fraction, $\mathbf{T}^{(\mathsf{f})} = \left[\mathbf{I} + \mathbf{P}\,\left(\mathbf{L}^{(\mathsf{f})} - \mathbf{L}^{(\mathsf{m})}\right)\right]^{-1}$ where $\mathbf{P}$ is Hill's tensor and $I_{ijkl} = \frac{1}{2}(\delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk})$.

The modeling of the effective viscoplastic properties of the MMC relies here on Dvorak's Transformation Field Analysis[3]. The resulting macroscopic model contains no fitting parameter and explicitly includes microstructural information such as the volume fraction of the fibers, their aspect ratio and their orientation distribution function.

In the case of two-phase materials, the final form of the model reads:

$$\boldsymbol{\Sigma} = \mathbf{L}^{\mathsf{eff}}(T)\,\mathbf{E}_{\mathsf{e}} = \mathbf{L}^{\mathsf{eff}}(T)\,(\mathbf{E} - \mathbf{E}_{\mathsf{vp}} - \mathbf{E}_{\mathsf{th}}) \tag{5}$$

$$\mathbf{E}_{\mathsf{th}} = c^{(\mathsf{m})}(\mathbf{B}^{(\mathsf{m})})^{\mathsf{T}}\,\boldsymbol{k}^{(\mathsf{m})}(T - T_{\mathsf{ref}}) + c^{(\mathsf{f})}(\mathbf{B}^{(\mathsf{f})})^{\mathsf{T}}\,\boldsymbol{k}^{(\mathsf{f})}(T - T_{\mathsf{ref}}) \tag{6}$$

$$\mathbf{E}_{\mathsf{vp}} = \frac{3}{2}c^{(\mathsf{m})}\left\langle \frac{J_2\left(\mathbf{B}^{(\mathsf{m})}\,(\boldsymbol{\Sigma}-\mathbf{X})\right) - \sigma_y(T)}{\eta(T)} \right\rangle^{n(T)} \frac{(\mathbf{B}^{(\mathsf{m})})^{\mathsf{T}}\mathsf{dev}\left(\mathbf{B}^{(\mathsf{m})}\,(\boldsymbol{\Sigma}-\mathbf{X})\right)}{J_2\left(\mathbf{B}^{(\mathsf{m})}\,(\boldsymbol{\Sigma}-\mathbf{X})\right)} \tag{7}$$

$$\boldsymbol{X} = \mathcal{H}\;\mathbf{E}_{\mathsf{vp}} - \mathcal{K}\,(T - T_{\mathsf{ref}}) + \mathcal{U}\;\boldsymbol{a} \quad \text{and} \quad \dot{\boldsymbol{a}} = \dot{\mathbf{E}}_{\mathsf{vp}} - \dot{P}\,\frac{\xi(T)}{c^{(\mathsf{m})}}\,\boldsymbol{a} \tag{8}$$

where $\boldsymbol{\Sigma}$ and $\mathbf{E}$ are respectively the macroscopic stress and strain, the fourth-order tensor $\mathbf{B}$ is the stress concentration factor, T is for the transpose operator, P is the cumulated plastic strain defined through $\dot{P} = \sqrt{\frac{2}{3}\dot{\mathbf{E}}_{\mathsf{vp}}\,\mathcal{Q}\,\dot{\mathbf{E}}_{\mathsf{vp}}}$ where $\mathcal{Q} = (\mathbf{B}^{(\mathsf{m})})^{\mathsf{T}}\,\mathbf{K}\,\mathbf{B}^{(\mathsf{m})}$ and $\mathbf{K}$ is the projector on deviators, $\mathbf{X}$ and $\boldsymbol{a}$ are kinematic variables, and

$$\mathcal{H} = \frac{1}{c^{(\mathsf{m})}}(\mathbf{B}^{(\mathsf{m})})^{-1}(\mathbf{I} - \mathbf{B}^{(\mathsf{m})})((\mathbf{L}^{(\mathsf{m})})^{-1} - (\mathbf{L}^{(\mathsf{f})})^{-1})^{-1}\mathbf{C}^{(\mathsf{m})}(\mathbf{B}^{(\mathsf{m})})^{-\mathsf{T}} \tag{9}$$

$$\mathcal{K} = (\mathbf{B}^{(\mathsf{m})})^{-1}(\mathbf{I} - \mathbf{B}^{(\mathsf{m})})((\mathbf{L}^{(\mathsf{m})})^{-1} - (\mathbf{L}^{(\mathsf{f})})^{-1})^{-1}(\boldsymbol{k}^{(\mathsf{f})} - \boldsymbol{k}^{(\mathsf{m})}) \tag{10}$$

$$\mathcal{U} = \frac{2}{3c^{(\mathsf{m})}} H(T)(\mathbf{B}^{(\mathsf{m})})^{-1}\,(\mathbf{B}^{(\mathsf{m})})^{-\mathsf{T}} \tag{11}$$

where the fourth-order tensor $\mathbf{C}^{(\mathsf{m})}$ is a corrective factor similar to, but different from, the corrective factor introduced by Chaboche et al[4] in order to correct the predictions of the standard TFA model which are known to be too stiff.

3 NUMERICAL IMPLEMENTATION OF THE MODEL

The implicit backward Euler method was used to integrate in time the constitutive model. Besides the thermomechanical state at time t_j, the variables prescribed at time t_{j+1} are $\Delta\mathbf{E}$, T_{j+1}, and all the quantities depending thereon ($\mathbf{L}^{(\mathsf{m})}$, $\Delta\mathbf{E}_{\mathsf{th}}$, etc.). To update the mechanical state, we first solve the system $\mathbf{r} = \{r_{\mathsf{e}}, r_{\mathsf{a}}, r_{\mathsf{p}}\} = \{0,0,0\}$ for $\mathbf{u} = \{\Delta\mathbf{E}_{\mathsf{e}}, \Delta\boldsymbol{a}, \Delta P\}$, where

$$r_{\mathsf{e}} = \Delta\mathbf{E} - \Delta\mathbf{E}_{\mathsf{e}} - \Delta\mathbf{E}_{\mathsf{vp}} - \Delta\mathbf{E}_{\mathsf{th}} \tag{12}$$

$$r_{\mathsf{a}} = \Delta\boldsymbol{a} - \Delta\mathbf{E}_{\mathsf{vp}} - \Delta P\,\frac{\xi(T)}{c^{(\mathsf{m})}}\,\boldsymbol{a}_{j+1} \tag{13}$$

$$r_{\mathsf{p}} = \Delta P - \; c^{(\mathsf{m})}\Delta t\left[\frac{J_2[\mathbf{B}^{(\mathsf{m})}\,(\mathbf{\Sigma}_{j+1} - \mathbf{X}_{j+1})] - \sigma_y(T_{j+1})}{\eta(T_{j+1})}\right]^{n(T_{j+1})} \tag{14}$$

Once the system is solved, $\mathbf{X}$ and $\mathbf{\Sigma}$ are updated using (5) and (8).

To solve the system, use is made of the Jacobian $d\mathbf{r}/d\mathbf{u}$—a byproduct of which is the consistent tangent operator $\partial\Delta\mathbf{\Sigma}/\partial\Delta\mathbf{E}$— given by

$$\frac{d\mathbf{r}}{d\mathbf{u}} = \begin{pmatrix} \mathbf{I} + \Delta P\,\mathcal{N}\,(\mathbf{L}^{\mathsf{eff}} + \mathcal{H}) & \Delta P\,\mathcal{N}\,\mathcal{U} & N \\ -\,\Delta P\,\mathcal{N}\,(\mathbf{L}^{\mathsf{eff}} + \mathcal{H}) & \left(1 + \,\Delta P\,\frac{\xi(T)}{c^{(\mathsf{m})}}\right)\mathbf{I} + \,\Delta P\,\mathcal{N}\,\mathcal{U} & -N + \frac{\xi(T)}{c^{(\mathsf{m})}}\,\boldsymbol{a} \\ -\,c^{(\mathsf{m})}\;\;\Delta t\,g\,N\,(\mathbf{L}^{\mathsf{eff}} + \mathcal{H}) & -\,c^{(\mathsf{m})}\;\;\Delta t\,g\,N\,\mathcal{U} & 1 \end{pmatrix} \tag{15}$$

where

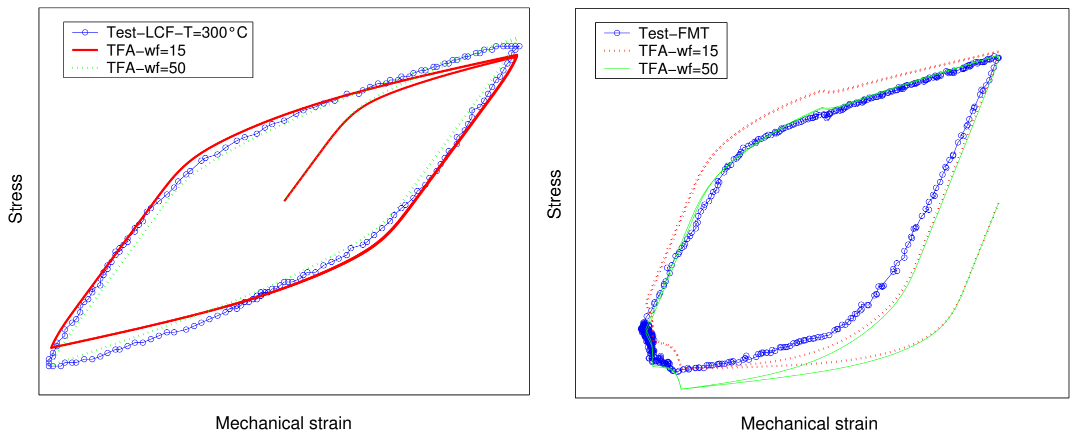
$$g = \frac{n(T)}{J_2[\mathbf{B}^{(\mathsf{m})}\,(\mathbf{\Sigma} - \mathbf{X})] - \sigma_y(T)}\;\left[\frac{J_2[\mathbf{B}^{(\mathsf{m})}\,(\mathbf{\Sigma} - \mathbf{X})] - \sigma_y(T)}{\eta(T)}\right]^{n(T)} \tag{16}$$

$$N = \frac{3}{2}\frac{\mathcal{Q}(\mathbf{\Sigma} - \mathbf{X})}{\sqrt{\frac{3}{2}(\mathbf{\Sigma} - \mathbf{X})^{\mathsf{T}}\,\mathcal{Q}\,(\mathbf{\Sigma} - \mathbf{X})}} \quad \text{and} \quad \mathcal{N} = \frac{1}{J_2[\mathbf{B}^{(\mathsf{m})}\,(\mathbf{\Sigma} - \mathbf{X})]}\left[\frac{3}{2}\mathcal{Q} - N\,N^{\mathsf{T}}\right] \tag{17}$$

4 APPLICATIONS

Predictions of the TFA-based model are compared with those of two kinds of tests performed at the Materials Center of École des Mines de Paris: (I) tension-compression isothermal low cycle fatigue tests at (i) 300°C, (ii) a strain ratio $R = 0$, and (iii) a test frequency of 0.1Hz (called hereafter, for short, LCF tests), and (II) strain and temperature controlled thermomechanical fatigue tests (called hereafter, for short, TMF tests), the range of temperature being 100℃ to 300℃.

The figure compares the stabilized cycle predicted by the model with the measured experimental data for an LCF test (left plot) and for an TMF test (right plot). The computations were carried out for two aspect ratios (15 and 50) because the actual aspect ratio is not well known.



The stabilized cycle of the LCF test is very well predicted, particularly in the case of the aspect ratio of 50. For the TMF test, the predictions are good.

5 CONCLUSION

A TFA-based constitutive model has been presented for describing the thermoviscoplastic macroscopic behavior of MMCs under complex thermomechanical loadings. The model has been implemented in the finite element code ABAQUS through a UMAT subroutine. The relative simplicity of the micro-macro model made possible an analytical derivation of the consistent tangent operator. The predictions of the model compared well with experiments, both under isothermal and anisothermal conditions.

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