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# FUZZYFICATION OF CHEN MODEL OF PLASTICITY OF CONCRETE

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**Summary.** *In this paper, it is attempted to introduce the effect of uncertainty to the Chen model of plasticity. For this purpose the material parameters are expressed in the form of fuzzy numbers and a suitable concept of material model fuzzification is proposed. The method of a fuzzy analysis is explained and shown in an illustrative example.*

## 1 INTRODUCTION

Material modeling is rather subjective in its nature. Ordered increasingly with respect to the cost of input information acquisition, the comparison shows that the deterministic analyses are the cheapest, however their results are of limited valididty; on the other end the probabilistic analyses provide the designer with extensive information including the distribution of the sought quantities, however, the input data acquisition is really expensive and in some cases, such as the design of unique structures, its use is irrelevant due to the lack of knowledge about the input parameters. In those cases, any pertinent estimation of the distribution of the input quantities is valuable and it can be incorporated into the modeling in terms of the possibility theory. The fuzzy calculus, which makes use of Zadeh's extension principle[1] applied to the fuzzy numbers, represents the basis of the proposed fuzzification concept. Another attempt of a strictly fuzzy-set-theory-based derivation of plasticity theory was presented by Klisinski[2], while de Lima et al. in their paper[3] show a comprehensive comparison between the probability and fuzzy approaches in relation to a subsoil elastoplastic analysis.

In the proposed concept, all material parameters considered in the Chen model of plasticity[4] are regarded as fuzzy numbers and therefore the entire analysis is performed with help of the fuzzy arithmetic. The result of such an analysis is in the form of fuzzy numbers, which compared with the deterministic approach provides extra information on the possible distribution of the investigated quantities, which corresponds to the distribution of the input parameters.

## 2 FUZZY NUMBER

The notion of a fuzzy number arises from the experience of the everyday life when many phenomena which can be quantified are not characterized in the terms of absolutely precise numbers.

Fuzzy numbers are fuzzy sets which are defined on the set of real numbers. Their membership function assigns the degree of 1 to the central, also called nominal, modal or mean, value and lower degrees to other numbers which reflect their proximity to the central value according to the used membership function. The membership function should thus decrease from 1 to 0 on both sides of the central value. Such fuzzy sets are called fuzzy numbers. An example of a fuzzy number is shown in Fig. 1, where $\mu$ represents the

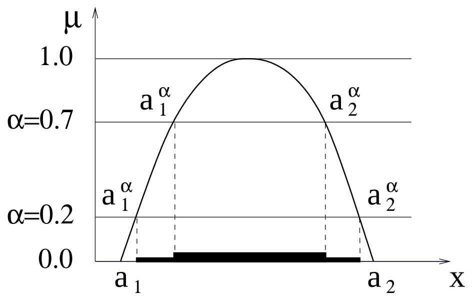


Figure 1: Normal fuzzy number and its $\alpha$-cuts

membership function and $a_1$ and $a_2$ stand for two real numbers on the real axis. The intervals defined for a specific value of the membership function, e.g. $\alpha = 0.7$, represent the so-called $\alpha$-cuts. A fuzzy number can be equally expressed by either a nominal value and a membership function on each side of the nominal value or by a set of $\alpha$-cuts.

## 3 FUZZYFICATION OF CHEN MODEL OF PLASTICITY

Chen model of plasticity is a three-parameter model for concrete displaying isotropic hardening[4] developed for description of the elastoplastic behavior of concrete, which is characterized by varying stress-strain characteristic under compression and tension. Therefore, two functions were proposed for each of the loading surfaces, in the compression-compression region and in the tension-tension or tension-compression regions. The following equations describe the initial and ultimate yield surfaces, $f_0^c$ and $f_u^c$, in the compression-compression region

$$f_0^c(\sigma) = J_2 + \frac{A_0}{3} I_1 - \tau_0^2 = 0, \tag{1}$$

$$f_u^c(\sigma) = J_2 + \frac{A_u}{3} I_1 - \tau_u^2 = 0, \tag{2}$$

where $A_0$, $\tau_0$, $A_u$ and $\tau_u$ are material constants which can be determined from simple tests. They are determined as functions of the ultimate stresses under uniaxial compression, $f_c$, and under equal biaxial compression, $f_{bc}$, and of the initial yield stresses under similar conditions, $f_{yc}$ and $f_{ybc}$, by the following relations

$$A_0 = \frac{f_{ybc}^2 - f_{yc}^2}{2f_{ybc} - f_{yc}}, \quad \tau_0^2 = \frac{f_{yc} f_{ybc}(2f_{yc} - f_{ybc})}{3(2f_{ybc} - f_{yc})}, \tag{3}$$

$$A_u = \frac{f_{bc}^2 - f_c^2}{2f_{bc} - f_c}, \quad \tau_u^2 = \frac{f_c f_{bc}(2f_c - f_{bc})}{3(2f_{bc} - f_c)}. \tag{4}$$

All entries in the Eqs. (3) and (4) are fuzzy number, e.i. the material constants in Eqs. (1) and (2) are also fuzzy numbers. The calculation is then performed through fuzzy arithmetic operation, which are based on the extension principle[1], on the fuzzy numbers which are expressed by the $\alpha$-cut representation. Attention should be paid to the dependencies present throughout the calculation so that the lower and upper bounds at each $\alpha$-cut do not spread beyond all limits. That means in, e.g., Eg. (4), in order to obtain the maximum possible value of $A_u$ at an $\alpha$-cut, one should not consider the upper bound of $f_c$ in the numerator and the lower bound of $f_c$ in the denominator at an $\alpha$-cut at one time. That would be unrealistic.

## 4 ILLUSTRATIVE EXAMPLE

In this example, the standard uniaxial compression test performed with a standard cylindrical specimen ($\phi = 100$ mm, height = 200 mm) is simulated. The material strength characteristics, present in Eqs. (3) and (4), are considered as shown in Table 1. Due to

| $f_c$ | $f_{yc}$ | $f_t$ | $f_{yt}$ | $f_{bc}$ | $f_{ybc}$ |
|---|---|---|---|---|---|
| 30 | 18 | 2.7 | 1.6 | 34.8 | 21 |

Table 1: Material strength characteristics in [MPa]

the lack of knowledge about the material characteristics, it is assumed that the values can vary by ± 10 %, same as for the value of the modulus of elasticity (nominal value of 33,000 MPa).

The computed compressive force - displacement diagram is shown in Fig. 2 where the possible distribution is described by the red line, which represents the most possible shape of the diagram, and the black and green lines, which represent the lower and upper bounds. It also should be noted that the realistic curve should be present within the limits given by the lower and upper bounds.

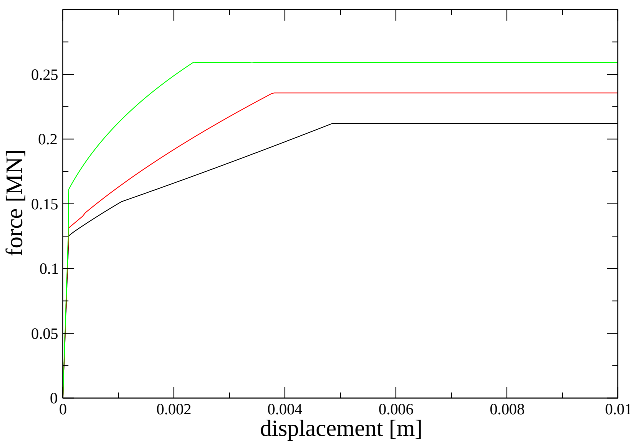


Figure 2: Force-displacement diagram

## 5 CONCLUSIONS

A concept of fuzzification of a material model is presented and it is applied to the Chen model of plasticity. This concept represents a tool which can help to reflect the extra information on material characteristics obtained from, e.g. field experts, in the results of elasto-plastic analyses. The advantage is the relatively easy implementation and comprehensive form of the results expressed as fuzzy numbers.

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