VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D. R. J. Owen (Eds)

MESOSTRUCTURAL NUMERICAL ANALYSIS OF TRABECULAR BONE ANISOTROPY

Olga Roa[*], Carlos M. López[†], Ignacio Carol[†]

[*]Matemáticas y Física, UNET (Univ. Nac. Exp. del Táchira), Venezuela

[†] School of Civil Engineering (ETSECCPB), Technical Univ. of Catalonia (UPC)
Jordi Girona 1-3, E-08034 Barcelona, Spain

email: olga.roa@upc.edu, carlos.maria.lopez@upc.edu, *ignacio.carol@upc.es*

Keywords: Plasticity, interface elements, trabecular bone, fracture, anisotropy

1 INTRODUCTION

Trabecular bone exhibits a heterogeneous random structure of beams and plates moderately oriented (trabecular architecture). The technical difficulties when performing experimental tests, has recently lead to the use of numerical analysis mainly based on the Finite Element Method. This study focuses on the mesostructural analysis of such architectures. Computer routines been developed in order to generate the anisotropic bone trabecular structure and corresponding finite element meshes, starting from average geometric parameters such as porosity and fabric tensor. The model incorporates fracture behavior using zero-thickness interface elements. Results of numerical tests under uniaxial tension of anisotropic specimens oriented at various angles are presented. All these calculations are repeated for various porosity values.

2 MESOSTRUCTURAL MODELING

On the basis of a model firstly developed and verified for concrete specimens[1], the trabecular bone specimen has been represented in 2D by means of an irregular structure of voids on and a solid matrix representing the bone tissue itself. The matrix of internal material structure is explicitly represented by triangular elements with linear elastic behavior, and interfaces are inserted between the predetermined standard elements of the continuum media, along the main potential crack paths to make it possible to study the nonlinear behavior and failure mechanisms. Interface behavior is formulated in terms of the normal and shear components of stresses (tractions) on the interface plane, $\sigma = [\sigma_N, \sigma_T]^t$, and corresponding relative displacements $u = [u_N, u_T]^t$ (t = transposed). The model is based on the theory of elasto-plasticity, and also includes some concepts of Fracture Mechanics. More details can be found in [1,2]. The generation of geometry is based on Voronoï/Delaunay polygonization, starting from an initially regular set of points which are randomly perturbed within small “perturbation boxes”. The anisotropy is tackled by means of a connection and an extension of the polygons in preferential directions. Subsequently, each polygon shrinks to end up as an irregular void structure. The solid material is discretized with conventional finite elements. In order to achieve a good representation of the anisotropy of the trabecular bone, the Mean

Intercept Length (MIL) concept is used, which represents a mean of inter-trabecular distance measured along a line and related with the inclination angle of the line to which the measure was done. Whitehouse[3] observed that, when plotting MIL values in a polar 2D diagram, the point distribution represents an ellipse (or the equivalent second order tensor). The relation between principal lengths of the ellipse is representative of the specimen anisotropy. More details can be found in [4].

3 RESULTS OF SPECIMENS "CUT" AT DIFFERENT ANGLES

The apparent elastic modulus and strength of a given trabecular specimen subject to uniaxial tension depends on the direction of the applied load and considerable variations are present with regard to values of the main geometrical parameters like porosity and fabric tensor [5,6]. To analyze those effects, six square specimens cut from the same larger anisotropic architecture but different orientations (at 15° intervals) in relation to the horizontal axis, were analyzed in elastic and non-linear range up to peak strength. The process was repeated for 5 different values of porosity (constant fabric relation H_1/H_2 0.7 approx.). The material parameters were selected to adjust the best global approximation to some reference experimental results.

Fig. 1 shows the polar diagrams of uniaxial elastic modulus (left) and peak stress (right), as a function of the orientation angle. In both diagrams the results of first quadrant correspond to loading along transversal orientation (horizontal), whereas the second quadrant corresponds to loading along longitudinal direction (vertical) of specimen. Fig.1 (left) shows as the elastic modulus increases for equal porosity values from horizontal to vertical direction, in agreement to principal values of fabric tensor. Also, as porosity decreases, the module increases in each orientation, although the relation between modules at 90° and 0° decreases in relative terms. In terms of peak stresses (Fig.1 right) the situation is a little different and a non-smooth transition is observed: whereas it value is more or less constant for a range of angles close to the lower principal direction (approximately circular envelope), in the zone close to the higher principal stiffness, a different “ellipse-type” envelope is obtained.

Fig. 2 depicts deformed meshes at failure (peak load) for specimens with porosity of 0.44 (up) and 0.74 (down), and three different load orientations (left to right). The behavior obtained for the lower porosity mesh is similar to that of a continuum medium, with displacement and Poisson effect at principal directions of fabric tensor, and shear displacement perpendicular to load direction for other orientations, as we can see in the mesh at 45°.

For the higher porosity mesh, a more pronounced localization of peak deformations is observed. In principal directions the deformation seems well aligned with axes, whereas for some other angles (see figure for 45°) an inclined localized band appears where trabeculaes rotate towards the load direction, whereas the rest of the specimen remains practically undeformed.

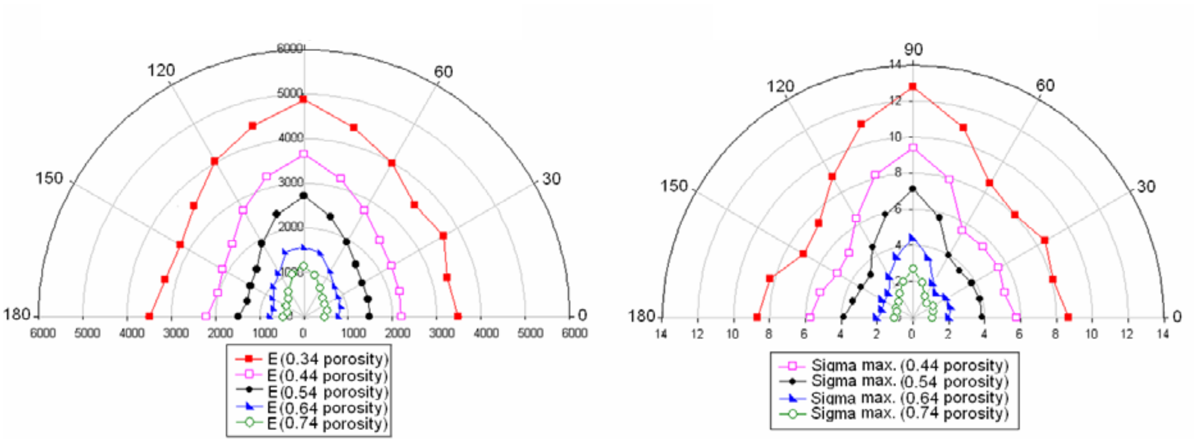

Figure 1. Polar diagrams for apparent elastic modulus (left) and peak stress (right) vs. angle orientation for different porosity values.

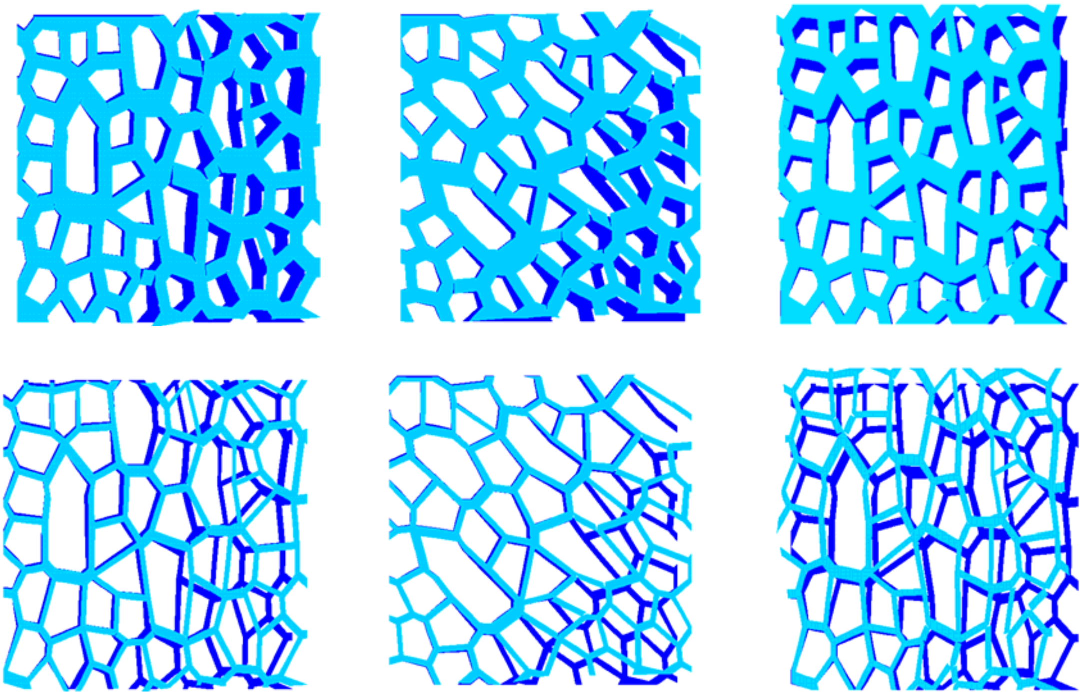

Figure 2. Deformed meshes for porosity values of 0.44 (up) and 0.74 (lower) for three different rotation angles: 0° (left), 45° (medium) and 90° (right).

4 CONCLUDING REMARKS

A mesostructural model initially developed and verified for concrete has been adapted to trabecular bone specimens. Generation of 2D anisotropic geometries based on Voronoi/Delaunay theory allow to simulate the bone porous structure reproducing desired values of average geometric features such as porosity and fabric tensor. Numerical simulations of uniaxial tension have been performed in anisotropic meshes obtained from the same mesh cut with different orientations of main axes of fabric tensor and for different porosity values. Interesting results from elasticity modulus and peak stress (strength) dependency on orientation have been obtained, as well as crack distributions and deformed meshes.

Additional results not shown here have been obtained in the tension-tension and tension-compression quadrants for each of the porosity values analyzed, leading to a more complete view of orthotropic elastic and anisotropic failure properties of this material. The failure results have been also fitted with the Tsai-Wu criterion[7] to which they adjust satisfactorily [4,8].

ACKNOWLEDGEMENTS

The first author wants to acknowledge the support received from UNET (Venezuela). The second author thanks MEC (Madrid, Spain) for his position funded under the "Ramón y Cajal" program. The research grant MAT2003-02481 also from MEC is also great fully acknowledged.

REFERENCES

[1] C.M. López, *Microstructural analysis of concrete fracture using interface elements. Application to various concretes* (In Spanish). Doctoral Thesis. Universitat Politecnica de Catalunya. ETSECCCP-UPC, E-08034 Barcelona, Spain, (1999).

[2] I. Carol, C. M. López and O. Roa, Micromechanical analysis of quasi-brittle materials using fracture-based interface elements, *Int. J. Numer. Meth. in Engrg*. Vol **52**, 193-215 (2001).

[3] J. Whitehouse, The quantitative morphology of anisotropic trabecular bone, *J. Microscopy*, Vol **101**, 153-168 (1974).

[4] O. Roa, *Análisis microestructural del comportamiento mecánico del hueso trabecular*, *concretes* (In Spanish). Doctoral Thesis. Universitat Politecnica de Catalunya. ETSECCCP-UPC, E-08034 Barcelona, Spain, (2004).

[5] H. A. Yuehuei, D. Robert, *Mechanical testing of bone and the bone-implant interface*, CRC Press, London (2000).

[6] S. C. Cowin, *Bone Mechanics*, CRC , Boca Raton-Florida (2001).

[7] S. Tsai and E. Wu, A general theory of strength for anisotropic materials, *J. Composite Materials*, Vol. **5**, 58-80 (1971).

[8] C.M. López, O. Roa and I. Carol, Caracterización geométrica, modelos micromecánicos y anisotropía del hueso trabecular. In C.A. Mota Soares, et al., editor, *Metodos Computacionais in Engenharia - (Proc. of the Int. Conf. in Lisbon 31/May- 2/Jun)*. APMTAC–SEMNI. In CD-ROM, ISBN 972-49-2008-9.