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MODELLING OF SHAPE MEMORY ALLOYS IN THE FINITE DEFORMATION RANGE CONSIDERING FATIGUE UNDER CYCLIC LOADING

Daniel Christ⋆**, Stefanie Reese**⋆

⋆ Institute of Mechanics, Department of Civil Engineering
Ruhr Univesity Bochum
Universitätsstr. 150, 44801 Bochum, Germany
E-mail: christ@nm.rub.de / reese@nm.rub.de, Web page: http://www.nm.rub.de

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Summary. *In this contribution we present a thermomechanical 3D model for SMA based on the concept of Helm & Haupt*[1] *which includes the effect of pseudoelasticity as well as the shape memory effect and the external two way effect. This model has been extended to large deformations and has been implemented into a FE code by using an implicit integration scheme. Additionally, we consider the aspect of mechanical fatigue of SMA under cyclic loading by introducing a damage function in the material model.*

1 INTRODUCTION

Shape memory alloys (SMA) can undergo phase transformation between a high-ordered austenite phase and a low-ordered martensite phase, as a result of changes in the temperature and the state of stress. Consequently, SMA exhibits several macroscopic phenomena not present in traditional materials. Two significant phenomena are the shape memory effect (SME) and the pseudoelasticity (PE). These unique features of SMA have found numerous applications in the mechanical, automotive, aerospace, electronic industries as well as in the medical field. The increasing use in commercially valuable applications have motivated a vivid interest in the development of accurate constitutive models to describe the thermomechanincal behaviour of SMA. The recent state of the art regarding the behaviour, the modelling and the applications of SMA is documented in the book of *Auricchio*[2] from 2001. However, model formulations which are capable to describe the various effects of SMA in a three-dimensional finite deformation framework are still lacking.

2 CONTINUUM MECHANICAL MODEL

As already mentioned, the proposed model is based in general on the concept of *Helm & Haupt*[1]. It is built upon a phenomenological model formulation for elastoplasticity. Its performance in combination with the FE method has been already investigated by *Christ & Reese*[3] for the case of small deformations. To reinforce the knowledge on the based model we refer to the above mentioned contributions[1,3]. At the beginning we introduce the deformation gradient $\mathbf{F}$ which can be decomposed into an elastic part and a transformation part ($\mathbf{F} = \mathbf{F}_e\,\mathbf{F}_t$) as it is applied in the classical crystal plasticity, too. Additionally the deformation gradient exhibiting the phase transformation $\mathbf{F}_t$ is split as well into an elastic part and a dissipative part ($\mathbf{F}_t = \mathbf{F}_{t_e}\,\mathbf{F}_{t_d}$). A second step is to introduce the elastic symmetric Cauchy-Green tensors $\mathbf{C}_e = \mathbf{F}_e^T\,\mathbf{F}_e = \mathbf{F}_t^{-T}\mathbf{C}\,\mathbf{F}_t^{-1}$ and $\mathbf{C}_{t_e} = \mathbf{F}_{t_e}^T\mathbf{F}_{t_e} = \mathbf{F}_{t_d}^{-T}\mathbf{C}_t\,\mathbf{F}_{t_d}^{-1}$. The total Helmholtz free energy Ψ consists also of an elastic and a transformation part $\Psi = \Psi_e\,(\mathbf{C}_e, z) + \Psi_t\,(\mathbf{C}_{t_e})$, whereas the elastic part depends on $\mathbf{C}_e$ and the martensitic volume fraction z and the transformation part only on $\mathbf{C}_{t_e}$. In order to derive the model in a thermodynamic frame we have to fulfil the second law of thermodynamics by means of the Clausius-Duhem inequality $-\dot{\Psi} + \frac{1}{2}\,\mathbf{S}\cdot\dot{\mathbf{C}} \geq 0$ which leads to

$$-\frac{\partial \Psi_e}{\partial \mathbf{C}_e}\cdot\dot{\mathbf{C}}_e - \frac{\partial \Psi_e}{\partial z}\,\dot{z} - \frac{\partial \Psi_t}{\partial \mathbf{C}_{t_e}}\cdot\dot{\mathbf{C}}_{t_e} + \mathbf{S}\cdot\frac{1}{2}\,\dot{\mathbf{C}} \geq 0. \tag{1}$$

The martensitic volume fraction z can be defined in dependence of the transformation strains, $z = ||\mathbf{E}_t||/\beta$, where $\mathbf{E}_t := \frac{1}{2}(\mathbf{C}_t - \mathbf{1})$ is the Green-Lagrange strain and β a material parameter describing the width of the hysteresis (see Fig. 1). By using this expression for z the rates appearing in (1) can be expressed by

$$\begin{aligned}
\dot{z} &= \frac{1}{\beta}\,\frac{\mathbf{E}_t}{||\mathbf{E}_t||}\cdot\frac{1}{2}\,\dot{\mathbf{C}}_t = (\frac{1}{\beta}\,\mathbf{F}_t\,\frac{\mathbf{E}_t}{||\mathbf{E}_t||}\,\mathbf{F}_t^T)\cdot\mathbf{d}_t \\
\dot{\mathbf{C}}_e &= -\mathbf{l}_t\,\mathbf{C}_e + \mathbf{F}_t^{-T}\,\dot{\mathbf{C}}\,\mathbf{F}_t^{-1} - \mathbf{C}_e\,\mathbf{l}_t \\
\dot{\mathbf{C}}_{t_e} &= -\mathbf{l}_{t_d}\,\mathbf{C}_{t_e} + \mathbf{F}_{t_d}^{-T}\,\dot{\mathbf{C}}_t\,\mathbf{F}_{t_d}^{-1} - \mathbf{C}_{t_e}\,\mathbf{l}_{t_d},
\end{aligned} \tag{2}$$

where the deformation rate tensor $\mathbf{d}_t$ is defined by $\mathbf{d}_t := \frac{1}{2}\,\mathbf{F}_t^{-T}\dot{\mathbf{C}}_t\,\mathbf{F}_t^{-1}$ and the known definitions $\mathbf{l}_t := \dot{\mathbf{F}}_t\,\mathbf{F}_t^{-1}$ and $\mathbf{l}_{t_d} := \dot{\mathbf{F}}_{t_d}\,\mathbf{F}_{t_d}^{-1}$ are introduced. By considering further tensor calculations and the symmetry of $\partial\Psi_e/\partial\mathbf{C}_e$ and $\partial\Psi_t/\partial\mathbf{C}_{t_e}$, equation (1) can be redefined as

$$(\mathbf{M} - 2\,\mathbf{F}_t^{-T}\,\mathbf{C}\,\mathbf{F}_t^{-1}\frac{\partial \Psi_e}{\partial \mathbf{C}_e})\cdot\frac{1}{2}\,\dot{\mathbf{C}} + (\mathbf{M}\underbrace{-\,\mathbf{X}_t - \mathbf{X}_z}_{-\mathbf{X}})\cdot\mathbf{d}_t + \mathbf{M}_t\cdot\mathbf{d}_{t_d} \geq 0. \tag{3}$$

Here, we have introduced the so-called Mandel stress tensors $\mathbf{M}$, $\mathbf{M}_t$ and the back stresses $\mathbf{X}_t$ and $\mathbf{X}_z$

$$\mathbf{M} := 2\,\mathbf{C}_e\,\frac{\partial \Psi_e}{\partial \mathbf{C}_e} \quad \text{and} \quad \mathbf{M}_t := 2\,\mathbf{C}_{t_e}\,\frac{\partial \Psi_t}{\partial \mathbf{C}_{t_e}}, \tag{4}$$

$$\mathbf{X}_t := 2\,\mathbf{F}_{t_e}\,\frac{\partial \Psi_t}{\partial \mathbf{C}_{t_e}}\,\mathbf{F}_{t_e}^T \quad \text{and} \quad \mathbf{X}_z := \frac{\Delta\Psi}{\beta}\,\mathbf{F}_t\,\frac{\mathbf{E}_t}{||\mathbf{E}_t||}\,\mathbf{F}_t^T. \tag{5}$$

The Mandel stress tensors are symmetric if we assume that Ψ_e and Ψ_t are isotropic functions of the right Cauchy-Green tensors $\mathbf{C}_e$ and $\mathbf{C}_{t_e}$, respectively. The back stress

$\mathbf{X}_z$ originates from the introduction of the volumetric martensite fraction z and depends via the energy difference between the austenitic and martensitic phase $\Delta\Psi = \Delta\hat{\Psi}(\theta)$ on the temperature. Physically, $\mathbf{X}_z$ can be interpreted as the middle stress plateau of the hysteresis rising its level by increasing temperature (see Fig. 1). The finally developed Clausius-Duhem inequality (3) is sufficiently satisfied by the relations

$$\begin{aligned} \mathbf{d}_t &= \dot{\lambda}\,\frac{\mathbf{M}^D - \mathbf{X}^D}{||\mathbf{M}^D - \mathbf{X}^D||} = \frac{\partial \Phi_{\mathsf{sma}}}{\partial \mathbf{M}} \\ \mathbf{d}_{t_p} &= \dot{\lambda}\,\frac{b}{\mu_t}\,\mathbf{M}_t^D \end{aligned} \tag{6}$$

for the evolution equations $\mathbf{d}_t$ and $\mathbf{d}_{t_d}$, respectively. The phase transformation function Φ_{sma} is given in the continuum mechanical context via

$$\Phi_{\mathsf{sma}} = ||(\mathbf{M} - \mathbf{X}_i - \mathbf{X}_z)^D|| - k. \tag{7}$$

The superscript "D" characterizes the deviator of a tensor and k is a material parameter describing the half height of the hysteresis (Fig. 1). As we use a von Mises-type phase transformation function, k can be equated with isotropic hardening in classical elastoplasticity. Further the Kuhn-Tucker conditions $\dot{\lambda} \geq 0$, $\Phi_{\mathsf{sma}} \leq 0$ and $\dot{\lambda}\,\Phi_{\mathsf{sma}} = 0$ have to be fulfilled. Up to this point the main considerations of the material model are stated but the authors want to refer to *Reese* & *Christ*[4] for a more detailed derivation of the present model and its integration algorithm for the implementation into the finite element method.

3 FATIGUE BEHAVIOUR

It is already known that the phase transformation in shape memory alloys is strongly influenced by cyclic loading. In repeated load cycles the stress plateaus decrease and residual stains increase (Fig. 2). *Wagner*[5] has investigated the behaviour of a NiTi wire under cyclic loading. He found out that the just described fatigue behaviour can be realistically modelled by means of an exponential function, given as

$$\Omega(N) = m_1 + m_2\, e^{-m\,N} \quad \text{with} \quad \Omega = \sigma_{\mathrm{AM}}, \sigma_{\mathrm{MA}} \text{ or } \varepsilon_{\mathrm{res}}, \tag{8}$$

where m, m_1 and m_2 are additional material parameters. Fig. 3 shows exemplarily the values of the phase transformation stress σ_{AM} (from (A)ustenite to (M)artensite) against the number of cycles at different maximum strains (2-6 %). The curve progression of the exponential function (solid line) is very close to the experimentally measured values. Similarly, σ_{MA} and $\varepsilon_{\mathrm{res}}$ can be fitted, too. Inserting these functions into the model and adjusting the material parameters correspondingly, the numerical results are in a very good accordance with the experimental tests (Fig. 4).

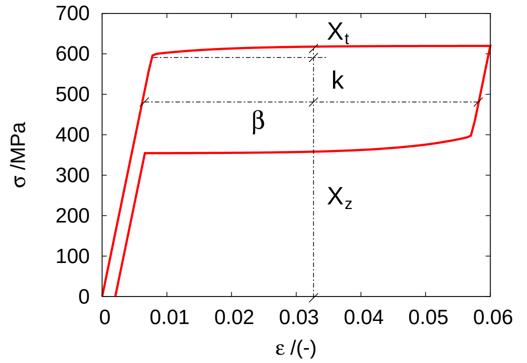

Figure 1: *Typical stress-strain hysteresis in pseudoelasticity. Definition of the different stress contributions*

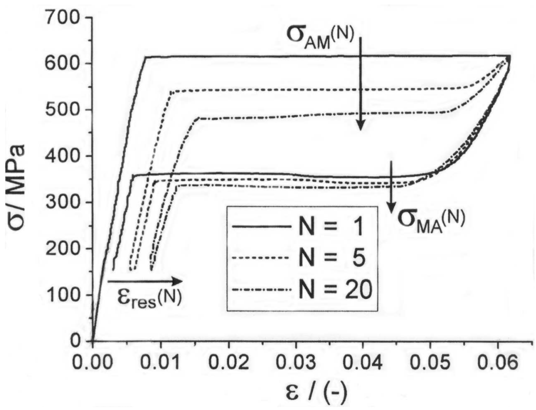

Figure 2: *Experimental results for a cyclic tension test after 1, 5 and 20 cycles*

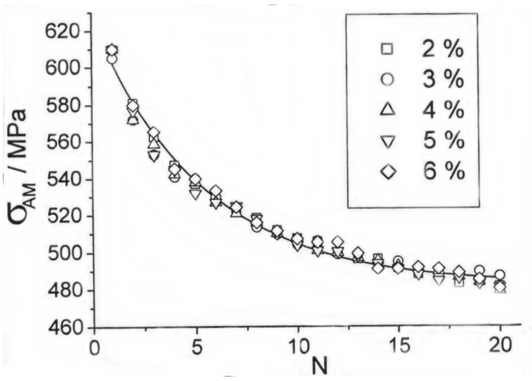

Figure 3: *Decreasing of the stress plateau (A → M) at different maximal strains*

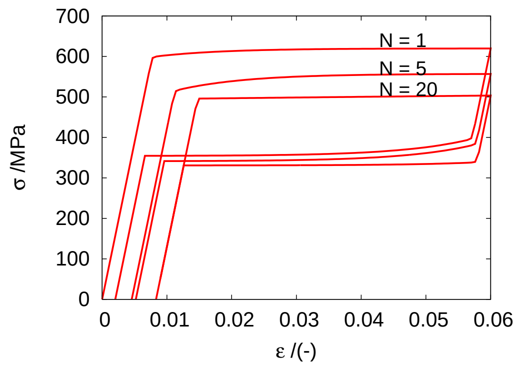

Figure 4: *Numerical results of a cyclic loading up to 20 cycles*

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