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FORMULATION AND IMPLEMENTATION OF GRADIENT ELASTICITY

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Summary. *Gradient elasticity models have been proposed in recent years in order to describe phenomena that cannot be described by classical elasticity models. In this contribution, a recently developed formulation of gradient elasticity will be presented that can be used in dynamics as well as in statics. It is an extension of the gradient elasticity model earlier suggested by Aifantis and coworkers, and it includes a length scale as well as a time scale. Special emphasis will be paid to the finite element implementation of the model. In particular, the implications of the so-called Ru-Aifantis theorem on the finite element implementation with* $\mathcal{C}^0$*-continuous shape functions will be studied.*

1 INTRODUCTION

Classical elasticity models, in which the stresses depend on the strains but not on higher-order derivatives, cease to be applicable if phenomena are to be described that are driven by processes on lower scales of observation. For example, wave dispersion is caused by the intrinsic inhomogeneity of any material but cannot be predicted by classical elasticity. Also the size-dependent mechanical properties of specimens are not reproduced by classical elasticity. Furthermore, in classical elasticity singularities in the stress and strain field may occur at the tips of a sharp crack, which are not realistic. In all these instances, gradient elasticity has been proven to be a viable alternative to classical elasticity. In its simplest format, the constitutive equation of gradient elasticity reads [1]

$$\sigma = D\left(\varepsilon - \ell^2\nabla^2\varepsilon\right) \tag{1}$$

where σ and ε are the Cauchy stress and the infinitesimal strain, D contains the usual elastic stiffness moduli, and ℓ is an additional parameter with the dimension of length. This intrinsic length scale may be thought of as a representative size of the underlying microstructure.

2 DYNAMICS — LENGTH SCALES AND TIME SCALES

Whereas Eq. (1) is suitable to avoid singularities in the strain field [1] and to describe size effects [4], for a realistic description of dispersive waves an extended format of gradient elasticity is needed. In [2, 3] a gradient elasticity model has been derived from a discrete lattice, whereby not only higher-order stiffness terms but also higher-order inertia terms are included:

$$\sigma = D\left(\varepsilon - \ell^2\nabla^2\varepsilon + \tau^2\ddot{\varepsilon}\right) \tag{2}$$

The higher-order inertia term is accompanied by an intrinsic time scale τ that sets the propagation characteristics of the high-frequency waves. The inclusion of higher-order inertia is crucial for a realistic description of wave dispersion.

3 IMPLEMENTATION — THE RU-AIFANTIS THEOREM

A disadvantage of gradient elasticity is that it requires a $\mathcal{C}^1$-continuous interpolation, since fourth-order displacement derivatives appear in the governing equations. However, it has been suggested in [1] that the original equations of gradient elasticity (1) enable an operator split. In particular, the following set of equations can be used as an alternative to the original equations:

$$\left\{\begin{array}{c} L^T D L u_c = 0 \\ u_g - \ell^2\nabla^2 u_g = u_c \end{array}\right. \tag{3}$$

where u_c and u_g are the displacements according to classical elasticity and gradient elasticity, respectively. The two separate sets in Equation (3) each only require $\mathcal{C}^0$-continuity, which constitutes a huge advantage. The influence of the boundary conditions (cf. [5]) and the extension towards the dynamics equation (2) will be addressed.

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