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PERTURBATION TECHNIQUE TO SOLVE PLASTICITY PROBLEMS

M. Assidi[a], H. Zahrouni[a], B. Braikat[b], N. Damil[b] and M. Potier-Ferry[a]

[a] L.P.M.M, U.M.R. CNRS 7554 I.S.G.M.P., Université de Paul Verlaine Ile du Saulcy, 57045 Metz, France

[b] Laboratoire de calcul scientifique en mécanique, Université Hassan II Mohammedia, Casablanca Maroc

E-Mail: assidi@lpmm.univ-metz.fr

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Summary. *We propose in this paper a new technique to regularize elastic-plastic constitutive law in view to apply the asymptotic numerical method (ANM) to computational plasticity.*

1 INTRODUCTION

The aim of this work is to present a new regularization technique to solve plasticity problems in the framework of the asymptotic numerical method (ANM). We recall that ANM is a family of techniques associating perturbation technique and finite element discretization which allows us to search solution branches in the shape of power series with a reduced number of decomposed matrices. A large bibliography about ANM can be found in[1]. Elastic plastic constitutive laws induce strong nonlinearities that one has to take into account in the numerical simulation of material forming process and combine two unilateral conditions. The first one concerns the transition from elastic domain to the plastic one and the second condition concerns the elastic unloading. An efficient ANM algorithm has not been proposed yet for the treatment of elastic unloading. Plastic behavior was taken into account within the framework of deformation theory of plasticity[2-3-4], for the Norton-Hoff model, for unilateral contact, or for problems coupling these nonlinearities[1].

2 A SMOOTH APPROXIMATION OF THE ELASTIC PLASTIC MODEL

In this study we adopt the flow theories with linear hardening and we take into account the elastic unloading. The basic idea is to replace the non smooth problem by a smooth one in such a way that solution curves can be expanded into power series[2-3]. The modified constitutive law is set in the following form:

$$\sigma = D:(\varepsilon - \varepsilon^p) \tag{1}$$

$$q^2 = \frac{3}{2}\sigma^d : \sigma^d + \eta_1^2 \sigma_y^2 \tag{2}$$

$$f = \frac{q - \sigma_e}{\sigma_y} \tag{3}$$

$$\sigma_e = \sigma_y + h\,\varepsilon_e^p \tag{4}$$

$$\dot{\varepsilon}^p = \dot{\lambda}\, n \tag{5}$$

$$\dot{\lambda} = \dot{\varepsilon}_c GH \tag{6}$$

$$\dot{\varepsilon}_c X = n : \dot{\varepsilon} \tag{7}$$

$$H(H - X) = \eta_2 \tag{8}$$

$$G = \frac{\eta_3}{f^2 + \eta_3(\frac{3}{2} + \frac{h}{2\mu})} \tag{9}$$

$$n = \frac{3}{2}\frac{\sigma^d}{q} \tag{10}$$

Where $\sigma, \varepsilon, \varepsilon^p, n, q, f, \sigma_e, \dot{\lambda}$ denotes respectively stress, strain, plastic strain, normal of the loading surface, von Mises stress, loading function, the yield stress and the plastic multiplier. μ, h, σ_y are the shear modulus, hardening modulus and the yield stress. Equation (2, 8, 9) involve regularization parameters η_1, η_2, η_3. These parameters are chosen significantly small in order that the smooth constitutive law is close to the exact plasticity model. The functions G in (9) and H in (8) describe the elastic plastic transition and the loading-unloading one.

3 HIGH ORDER SOLUTION TECHNIQUE

In computational structural mechanics, as well posed problem is defined by associating the constitutive law with the equilibrium equation and boundary conditions. Within the framework of asymptotic numerical method, the aforementioned problem is solved by an algorithm coupling a spatial discretization, the perturbation technique and a continuation procedure. Within this framework, the solution path is represented by truncated power series in the form:

$$U = U_0 + aU_1 + a^2U_3 + ... + a^N U_N \tag{11}$$

Where $U = (\sigma, \varepsilon, \varepsilon^p, ...)$ contains the unknowns of the constitutive law, U_0 is a starting values of the current path, N is a given truncation order and "a" is a suitable path parameter. A key point is the evaluation of the end point of the interval $a_{\max}$. Within the ANM this latter is automatically computed and usually evaluated by requiring that the last term of the truncated series is small enough with respect to the first one. For the plasticity problems, we propose to define the step length from the evolution of several unknowns as follows:

$$a_\sigma = \left(\delta \frac{\sigma_1}{\sigma_N}\right)^{\frac{1}{N-1}} \quad a_\varepsilon = \left(\delta \frac{\varepsilon_1}{\varepsilon_N}\right)^{\frac{1}{N-1}} \quad a_H = \left(\delta \frac{H_1}{H_N}\right)^{\frac{1}{N-1}} \quad a_G = \left(\delta \frac{G_1}{G_N}\right)^{\frac{1}{N-1}} \tag{12}$$

$$a_{\max} = \min(a_\sigma, a_\varepsilon, a_G, a_H)$$

The parameter δ in (12) is a small user parameter that permits to control the accuracy of the solution and the size of the step length. Classically, the ANM step length is defined from a single type of variable, as displacement or velocity fields[1]. With this new measure of the step length (12), there is a large probability to get an accurate solution of each equation in (1-10) along the step.

4 EVALUATION OF THE NUMERICAL PROCEDURE

In this section, numerical tests presented in order to evaluate the ability of the procedure to present a typical process (elastic, plastic, elastic unloading). We impose a strain in the shape of equation (13). A regularized relation between t and $\varepsilon(t)$ is assumed:

$$\left(\varepsilon-\varepsilon_m\frac{t}{T}\right)\left(\varepsilon-\varepsilon_m(2-\frac{t}{T})\right)=\eta_4\varepsilon_m^2 \tag{13}$$

In the numerical test, data are chosen as:

$E=2.10^5 MPa,\ \nu=0.3, \sigma_y=240 MPa,\ T=100s,\ \varepsilon_m=5.10^5\varepsilon_y, \varepsilon_y=\dfrac{\sigma_y}{E}, \eta_4=0.1.$ As for the regularization of the elastic plastic model, two cases will be considered: a weakly modified case $\eta_2=0.1$ $\eta_3=0.1$ table (1a) and a more strongly modified case $\eta_2=0.1$ $\eta_3=0.5$ table (1b). Many computations of this problem have been done for values the accuracy parameter δ smaller than (10^{-3}), for several orders N and for various values of the regularization parameters η_2, η_3 and η_4 . Some typical results are presented on figures (1a) and (1b) and on the tables (1a) and (1b). The response curves have been obtained in a correct and automatic manner in all the tested cases. Thus, the present procedure seems to be very robust. The figures established that the modification of the constitutive laws does not affect significantly the physics of the problem. Especially, the elastic modulus, the plastic slope and the unloading phase are predicted correctly. The difference between the smooth and the non smooth curves is restricted to the vicinity of the corners. The first corner (elastic to plastic) is more softened if η_3 is larger. The same holds for the second corner (unloading) with large η_2 .The convergence properties are quite similar to those obtained in various problems solved by ANM[1]. The number of steps decreases if the order N, the accuracy parameter δ or the regularization parameters increases (see table 1a and 1b). Note that no correction is needed throughout the computation contrary to the classical iterative algorithms which require iterations at two levels: to compute the stress at the integration point and to correct the residual of the equilibrium equation.

	$\delta=10^{-6}$	$\delta=10^{-4}$	$\delta=10^{-3}$
N=10	110	72	55
N=15	64	49	42
N=20	49	40	37
N=30	37	36	34

(a)

	$\delta=10^{-6}$	$\delta=10^{-4}$	$\delta=10^{-3}$
N=10	88	55	40
N=15	54	41	32
N=20	38	34	28
N=30	32	28	25

(b)

Table 1 : Number of steps versus the truncation orders

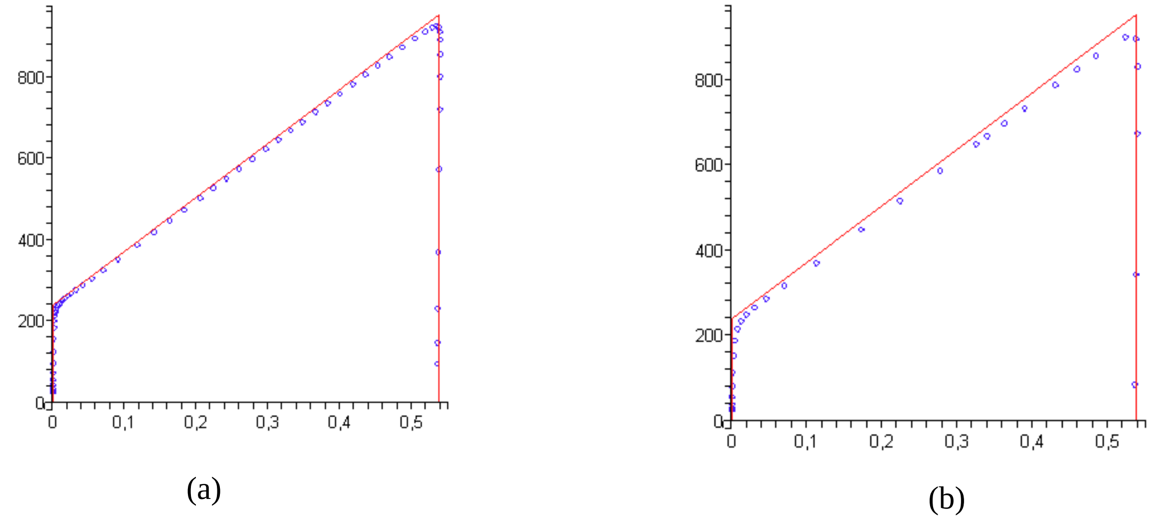

Figure 1: Stress-strain response: for (a) $N = 15, \delta = 10^{-7}, \eta_2 = \eta_3 = 0.1$ (66 steps) and (b) $N = 15, \delta = 10^{-3}, \eta_2 = 0.1, \eta_3 = 0.5$ (32 steps). The point denote the step ends of the asymptotic solution and the continue line represents the analytical solution.

5 CONCLUSIONS

A new manner to get smooth approximations of response curves within plasticity has been presented. As compared with previous works, the main innovation is the treatment of elastic unloading. This method can be easily applied in finite element framework and this could permit to introduce some advantages of ANM in computational plasticity, especially to define a robust algorithm to calculate response curves with abrupt changes of direction. The management of computation is often difficult in plasticity, because the problem involves two coupled unilateral conditions. An improved technique to define the step length was necessary to get a robust algorithm and no correction is needed throughout the computation contrary to the classical iterative algorithms which require iterations at two levels: to compute the stress at the integration point and to correct the residual of the equilibrium equation.

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