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DISCONTINUOUS NUMERICAL MODELING OF FRACTURE USING EMBEDDED DISCONTINUITIES

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Summary. *In this work, a strong embedded discontinuity approach is used to model fracture in quasi-brittle materials. In this approach, a true discontinuous displacement field is adopted. The discontinuities, arbitrarily located inside the finite element, are embedded in a pure discrete way, which is similar to the use of interface elements. Within each parent element, the additional displacements induced by the jumps correspond to rigid body motions. As a consequence, opposite to the extended finite element method (XFEM), it is not necessary to perform a numerical integration on each part of the parent element.*

1 INTRODUCTION

The kinematics of the *strong discontinuity approach* entails the description of a discontinuous displacement field across an internal boundary Γ_d; however, in previous strong discontinuity formulations, the displacement jumps are smeared over the entire parent element[1]. As a result, these models should not be considered within the framework of a discrete crack approach[2].

In the extended finite element method(XFEM)[3], a *true* discontinuous displacement field is adopted; however, the concept of embedded discontinuities is no longer addressed since the enriched nodes do not lie at the discontinuity.

Here, a pure discrete crack concept is followed such that the implementation of the strong discontinuity formulation is much similar to the implementation of a discrete crack model using interface elements. The most significant features may be summarized as follows: i) the internal boundary Γ_d is *embedded* in the parent element; ii) the jumps are obtained at *additional nodes* which are *located at* Γ_d; iii) these additional nodes can be considered either as *local* or *global*; iv) a *consistent variational formulation* is adopted, which is *symmetric* if the constitutive matrices are symmetric; v) a *non-homogeneous displacement jump field* is adopted within each parent element; vi) the approximated displacement field is *discontinuous* and vii) within each parent element, the additional displacements induced by the jumps correspond to *rigid body motions*. These two last properties are characteristic of all the discrete formulations adopting interface elements. Moreover, due to vii), the *orthogonality condition* between the admissible stress and enhanced strain spaces is fulfilled *exactly*[4]. From the numerical point of view, the consideration of rigid body motions also presents an advantage when compared to the extended finite element method , since it is not necessary to perform a numerical integration on each part of the parent element; in fact, in the present model, the energy evaluated in the continuum part of the element is solely due to the elastic strain energy, whereas the energy dissipated due to cracking is exclusively derived from the discontinuity Γ_d.

2 KINEMATICS OF A DISCONTINUITY

Consider a domain Ω, with boundary $\partial\Omega$, where a discontinuity surface Γ_d is supposed to exist The total displacement field is considered as the sum of a regular part $\hat{\mathbf{u}}$ on Ω and a discontinuous part corresponding to the displacement jump $[\![\mathbf{u}]\!]$, localized at the discontinuity surface Γ_d:

$$\mathbf{u}(\mathbf{x}) = \begin{cases} \hat{\mathbf{u}}(\mathbf{x}) + \tilde{\mathbf{u}}^+(\mathbf{x}) & \text{if } \mathbf{x} \in \Omega^+ \\ \hat{\mathbf{u}}(\mathbf{x}) + \tilde{\mathbf{u}}^-(\mathbf{x}) & \text{if } \mathbf{x} \in \Omega^-. \end{cases} \tag{1}$$

In equation (1), $\tilde{\mathbf{u}}$ is the additional displacement field due to the discontinuity jump $[\![\mathbf{u}]\!]$, such that:

$$[\![\mathbf{u}]\!] = \tilde{\mathbf{u}}^+ - \tilde{\mathbf{u}}^- \quad \text{at } \Gamma_d. \tag{2}$$

The total strain in the body is given by:

$$\boldsymbol{\varepsilon} = \boldsymbol{\nabla}^{\mathrm{s}}\mathbf{u} = \boldsymbol{\nabla}^{\mathrm{s}}\hat{\mathbf{u}} + \tilde{\boldsymbol{\varepsilon}} \quad \text{in } \Omega \setminus \Gamma_d, \tag{3}$$

where $(\cdot)^{\mathrm{s}}$ refers to the symmetric part of $(\cdot)$, the regular strain field is obtained from the continuous part of the displacement field and the enhanced strain is

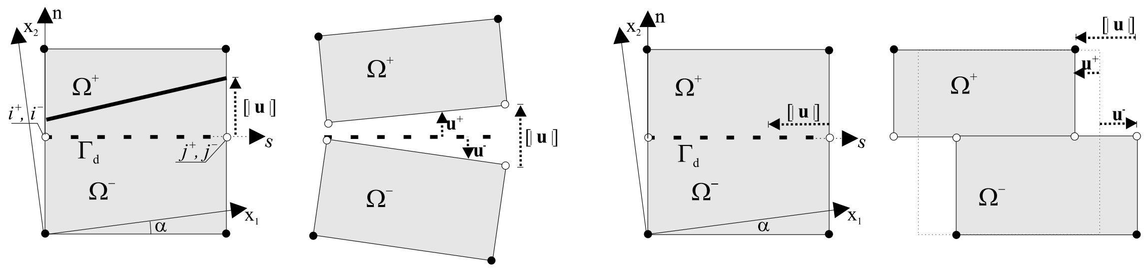
$$\tilde{\boldsymbol{\varepsilon}} = \begin{cases} \boldsymbol{\nabla}^{\mathrm{s}}\tilde{\mathbf{u}}^+(\mathbf{x}) & \text{if } \mathbf{x} \in \Omega^+ \\ \boldsymbol{\nabla}^{\mathrm{s}}\tilde{\mathbf{u}}^-(\mathbf{x}) & \text{if } \mathbf{x} \in \Omega^-. \end{cases} \tag{4}$$

3 FINITE ELEMENT APPROXIMATION

Consider a finite element discretisation of the 2D domain Ω. Assume that one element is crossed by a straight discontinuity Γ_d, which divides Ω in two sub-domains Ω^+ and Ω^-. A local frame (s,n) is introduced such that $s(\mathbf{x})$ is aligned with Γ_d and n is the normal to the discontinuity (fig. 1). Recall equations (1) and (2). For the sake of simplicity, assume that the jump $[\![\mathbf{u}]\!]$, $\tilde{\mathbf{u}}^+|_{\Gamma_d}$ and $\tilde{\mathbf{u}}^-|_{\Gamma_d}$ are linear functions of s, where

$$[\![\mathbf{u}]\!] = [\![\mathbf{u}\,(s(\mathbf{x}))]\!] = (\tilde{\mathbf{u}}^+ - \tilde{\mathbf{u}}^-)|_{\Gamma_d}. \tag{5}$$

In (fig.1), where $\hat{\mathbf{u}}$ is neglected for clarity, the total displacement field is depicted in two different situations: in (fig.1a) the discontinuity opens in the normal direction only, whereas in (fig.1b) the discontinuity represents a shear band undergoing sliding displacements.



a) tensile, normal (split) jump

b) shear, tangential (slip) jump

Figure 1: Displacement jump in a four node element crossed by a discontinuity

In the example above, the additional four nodes (i^+, i^- and j^+, j^-, each pair of nodes i and j initially coincide) are located at the intersection of Γ_d with the edges of the element. In matrix form, for each finite element e with n nodes, the following approximation of the displacement field is adopted:

$$\begin{aligned} \hat{\mathbf{u}}^e &= \mathbf{N}^e(\mathbf{x})\hat{\mathbf{a}}^e && \text{in } \Omega^e \setminus \Gamma_d^e \\ [\![\mathbf{u}]\!]^e &= \mathbf{N}_w^e\,[s(\mathbf{x})]\,(\mathbf{w}^{e+} - \mathbf{w}^{e-}) && \text{at } \Gamma_d^e \end{aligned} \tag{6}$$

where $\mathbf{N}^e$ contains the usual element shape functions, $\hat{\mathbf{a}}^e$ are the nodal degrees of freedom associated with $\hat{\mathbf{u}}^e$, $\mathbf{N}_w^e$ are the shape functions used to approximate the jumps $[\![\mathbf{u}]\!]^e$ and $\mathbf{w}^{e+}$ and $\mathbf{w}^{e-}$ are the degrees of freedom associated with $\tilde{\mathbf{u}}^{e+}|_{\Gamma_d^e}$ and $\tilde{\mathbf{u}}^{e-}|_{\Gamma_d^e}$, measured at nodes i^+, j^+ and i^-, j^-, respectively.

Simo and Rifai[4] assumed that S^h and $\tilde{\zeta}^h$ are L_2 orthogonal, where S^h and $\tilde{\zeta}^h$ are the admissible stress space and the admissible enhanced strain space, respectively. As a result, the work done by the stresses on the enhanced strains in an element is null. Applying this orthogonality condition to Ω^{e+} and Ω^{e-}, gives

$$\int_{\Omega^{e+}} (\nabla^s \tilde{\mathbf{u}}^{e+})^T : \boldsymbol{\sigma}^e d\Omega = \int_{\Omega^{e-}} (\nabla^s \tilde{\mathbf{u}}^{e-})^T : \boldsymbol{\sigma}^e d\Omega = 0. \tag{7}$$

In the present formulation, equation (7) is enforced, by imposing that the displacements $\tilde{\mathbf{u}}^{e+}$ and $\tilde{\mathbf{u}}^{e-}$ induce a null enhanced strain field:

$$\tilde{\boldsymbol{\varepsilon}}^{e+} = \nabla^s \tilde{\mathbf{u}}^{e+} = \mathbf{0} \text{ in } \Omega^{e+}, \qquad \tilde{\boldsymbol{\varepsilon}}^{e-} = \nabla^s \tilde{\mathbf{u}}^{e-} = \mathbf{0} \text{ in } \Omega^{e-} \tag{8}$$

Consequently, the additional displacement fields $\tilde{\mathbf{u}}^{e+}$ and $\tilde{\mathbf{u}}^{e-}$: i) must be evaluated separately in subdomains Ω^{e+} and Ω^{e-}, respectively, and ii) correspond to rigid body motions.

By means of the field approximations given in equations (6) the principle of virtual work leads to[5]

$$\mathbf{K}_{aa}^e d\hat{\mathbf{a}}^e = d\mathbf{f}_{ext}^e, \quad \mathbf{K}_{dd}^e \mathbf{w}^{e+} = d\mathbf{f}_{w,ext}^{e+}, \quad \mathbf{K}_{dd}^e \mathbf{w}^{e-} = d\mathbf{f}_{w,ext}^{e-} \tag{9}$$

The additional nodes can be taken as local, such as in previous embedded formulations, or global; using the latter option, continuity of the jumps at the discontinuities across the element boundaries is automatically enforced[6].

4 CONCLUSIONS

In this work, a strong embedded discontinuity approach is presented, which fits in the framework of a discrete crack approach. Similar to the discrete-interface approach, the element crossed by a discontinuity is divided into two subdomains, Ω^{e+} and Ω^{e-}; however, this separation involves no remeshing since these two subdomains are not considered as new elements. The additional degrees of freedom can be adopted as local, as done in previous embedded formulations, or global. In the latter case, continuity of the jumps across element boundaries is automatically fulfilled. The additional displacements due to the displacement jump at the localized discontinuity are transmitted to the regular nodes as rigid body motions; due to this fact, the enhanced strains are null and the orthogonality condition is fulfilled exactly on $\Omega^e \setminus \Gamma_d^e$: the admissible stress space and the admissible enhanced strain space are L_2 orthogonal in each parent element. As a result, the work done by the stresses on the enhanced strains is null. Finally, a consistent variational formulation is used, which is symmetric if the constitutive matrixes are symmetric.

REFERENCES

[1] J. Oliver, Modelling strong discontinuities in solid mechanics via strain softening constitutive equations. Part 1: Fundamentals, *International Journal for Numerical Methods in Engineering*, 39(21), 3575–3600, 1996.

[2] R. de Borst, G. N. Wells, L. J. Sluys, Some observations on embedded discontinuity models, *Engineering Computations*, 18(1–2), 241–254, 2001.

[3] N. Möes, J. Dolbow, T. Belytschko, A finite element method for crack growth without remeshing, *International Journal for Numerical Methods in Engineering*, 46(1), 131–150, 1999.

[4] J. C. Simo, M. S. Rifai, A class of mixed assumed strain methods and the method of incompatible modes, *International Journal for Numerical Methods in Engineering*, 29(8), 1595–1638, 1990.

[5] J. Alfaiate, L. J. Sluys, On the use of embedded discontinuities in the framework of a discrete crack approach, in *WCCMVI, Sixth International Congress on Computational Mechanics in conjunction with the second Asian-Pacific Congress of Computational Mechanics*, 2004.

[6] J. Alfaiate, A. Simone, L. J. Sluys, Non-homogeneous displacement jumps in strong embedded discontinuities, *International Journal of Solids and Structures*, 40(21), 5799–5817, 2003.