VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)

IMPOSING ESSENTIAL BOUNDARY CONDITIONS IN THE EXTENDED FINITE ELEMENT METHOD

N. Moës, E. Béchet, and M. Tourbier

GeM Institute
Ecole Centrale de Nantes - Université de Nantes - CNRS
1 Rue de La Noe, 44321 Nantes, France
e-mail: nicolas.moes@ec-nantes.fr

Key words: Extended finite element method, Dirichlet boundary conditions, lagrange multiplier

Summary. *This paper is devoted to the imposition of Dirichlet type conditions within the eXtended Finite Element Method. The X-FEM method allows one to easily model surfaces of discontinuity or domain boundaries on a mesh not necessarily conforming to these surfaces. Imposing Neumann type boundary condition on boundaries running through the elements is straightforward. It is not the case for Dirichlet type boundary conditions (or the limiting case of stiff boundary conditions). In this paper, we show that the strong imposition of Dirichlet conditions inside elements leads to locking and suggest a remedy.*

1 INTRODUCTION

Imposing Dirichlet type boundary condition in the finite element framework is quite straightforward. For instance, these can be imposed directly by L2 projection on the boundary or by using Lagrange multipliers. The proper choice of the Lagrange multiplier space and subsequent integration of the bilinear form is well understood, see for instance [1] or [2]. The key issue is the verification of the so-called LBB condition.

In the X-FEM, the difficulty arises from the fact that there is no unique way to impose Dirichlet boundary conditions. To illustrate this fact on a simple case, consider the mesh in figure 1. The domain of interest Ω and it's Dirichlet boundary Γ are not matched by the mesh. How can one impose a value, say zero, on Γ ? For instance, to have a zero value at point C, values at nodes A and B simply need to be opposite (we assume that Γ runs through the middle of the last layer of elements). A closer look shows that for the interpolation to meet zero on Γ, the layer of nodes right above Γ must be set to the same value and all the nodes below Γ must be set to the same opposite value. The normal flux of the interpolation is thus constant along Γ which is not really satisfactory.

This simple example demonstrates, first, that there is no unique way to impose strongly Dirichlet boundary conditions in the X-FEM and, second, that the normal flux of the resulting interpolation is quite poor on the boundary ("boundary locking" occurs).

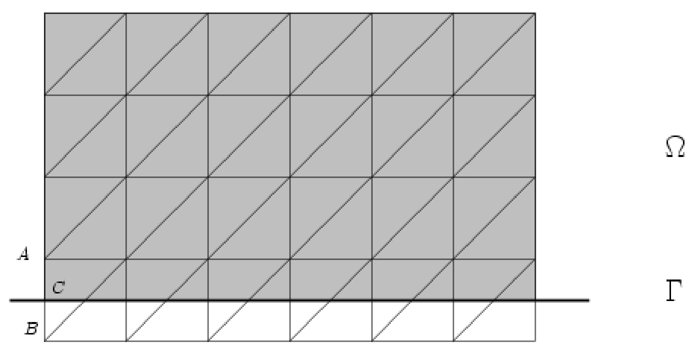

Figure 1: A domain of interest Ω (shaded area) located on a mesh not matching the Dirichlet boundary Γ of Ω.

The difficulty of choosing the proper Lagrange multiplier space to enforce interfacial constraints in the X-FEM framework was already observed in [3]. The authors showed that a naive construction of the Lagrange multiplier lead to oscillations.

To further illustrate the locking issue, we shall solve analytically the two-element scalar problem shown in figure 2. The nodal forces F_1 and F_2 are supposed to model the action of the upper elements. The Lagrange multiplier space is discretized over Γ by a linear interpolation parametrized by the λ_1, λ_2 and λ_3 coefficients. The inner space is discretized with the four "displacements" u_i, $i = 1, \ldots, 4$. The Lagrange multipliers solution is

$$\lambda_1 = F \tag{1}$$

$$\lambda_2 = F + \frac{1}{(1-e)}[F] \tag{2}$$

$$\lambda_3 = F - \frac{1}{(1-e)^2}[F] \tag{3}$$

where $F = F_1 + F_2$ and $[F] = F_1 - F_2$. We observe that the second and third Lagrange multiplier are blowing up when the interfaces reaches the bottom layer of elements. Especially λ_3 whose support drops to zero as e tends to one. Moreover, oscillation occur : the effect of $[F]$ is positive for λ_2 and negative for λ_3. The activation of three Lagrange multipliers in the preceeding example corresponds to a strong imposition of the Dirichlet condition and leads to locking. On the other hand, if we reduce the Lagrange multiplier space by imposing $\lambda_2 = (1-e)\lambda_1 + e\lambda_3$ (linear variation over Γ), we obtain

$$\lambda_1 = F + \left(\frac{2e^3 - 8e^2 + 9e - 4}{4e^5 - 16e^4 + 28e^3 - 32e^2 + 17e - 4}\right)[F] \tag{4}$$

$$\lambda_2 = (1-e)\lambda_1 + e\lambda_3 \tag{5}$$

$$\lambda_3 = F - \left(\frac{2e^3 - 8e^2 + 9e - 4}{4e^5 - 16e^4 + 28e^3 - 32e^2 + 17e - 4}\right)[F] \tag{6}$$

and the oscillations are gone. From this simple analysis, a strategy is designed in [4] to build a reduced Lagrange multiplier space in the general case. For instance, Figures 3 give the results for a sequence of non matching meshes on a scalar model problem already treated in [5] (the two first meshes of this sequence are shown in Figure4). The inf-sup parameter (numerical inf-sup test [6]) is given as well as the convergence of the errors (the upper of the three curves is the energy error, the middle curve is the L2 error on the flux and the bottom curve is the error on the imposed Dirichlet value). The inf-sup parameter is stable denoting no locking. More general cases are treated in [4].

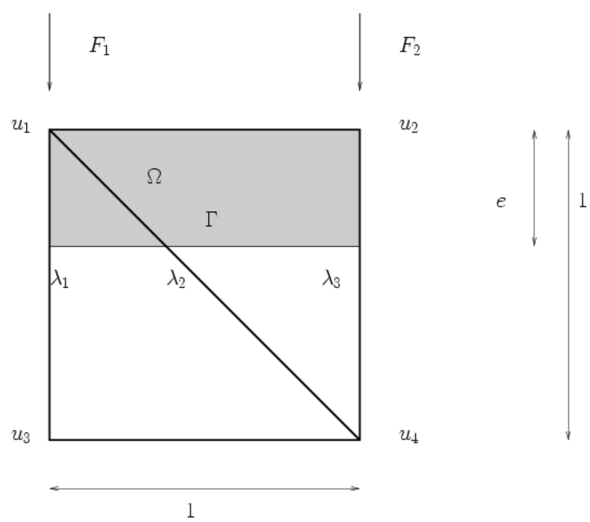

Figure 2: Analytical model of two elements.

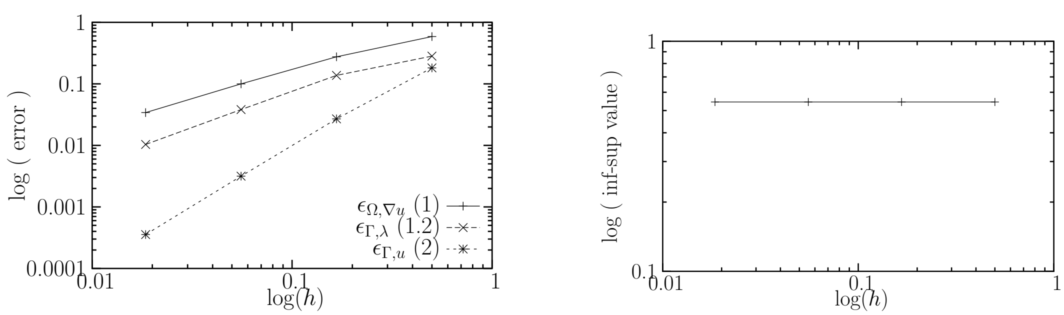

Figure 3: Results for the non matching (uniform) meshes case using of the reduced multiplier space : convergence of the errors and inf-sup parameter.

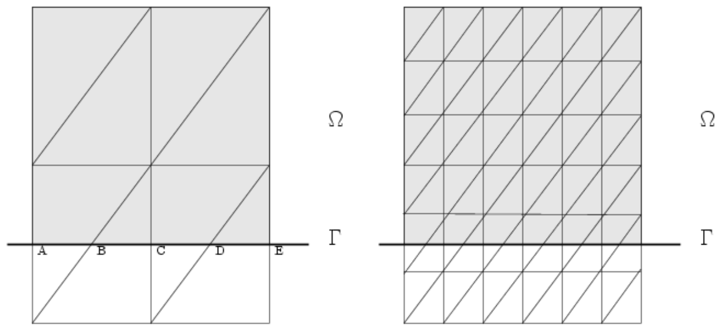

Figure 4: The first two meshes of a sequence of non matching structured meshes.

REFERENCES

[1] I. Babuška. The finite element method with lagrangian multipliers. *Numerische Math*, 20:179–192, 1973.

[2] H. Barbosa and T. Hughes. Finite element method with lagrange multipliers on the boundary. circumventing the babuska-brezzi condition. *Comp. Meth. in Applied Mech. and Engrg.*, 85(1):109–128, 1991.

[3] H. Ji and J.E. Dolbow. On strategies for enforcing interfacial constraints and evaluating jump conditions with the extended finite element method. *International Journal for Numerical Methods in Engineering*, 61:2508–2535, 2004.

[4] N. Moës, E. Béchet, and M. Tourbier. Imposing essential boundary conditions in the extended finite element method. *International Journal for Numerical Methods in Engineering*. Submitted.

[5] S. Fernández-Méndez and A. Huerta. Imposing essential boundary conditions in meshfree methods. *Comp. Meth. in Applied Mech. and Engrg.*, 193:1257–1275, 2004.

[6] N. El-Abbasi and K.J. Bathe. Stability and patch test performance of contact discretizations and a new algorithm. *Computers and Structures*, 79:1473–1486, 2001.