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RELIABILITY OF FRACTURING CONCRETE STRUCTURES AND CHALLENGES OF STOCHASTIC FINITE ELEMENT MODELING

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Summary. *The lecture identifies the need for several fundamental improvements in reliability concepts guarding against quasibrittle failures of structures made of concrete or other quasibrittle materials (fiber composites, ice, rock, etc.) and argues that such improvements should be much more profitable than improvements of accuracy of deterministic aspects of structural analysis.*

1 INTRODUCTION

During the last two decades, researches on quasibrittle failure have led to major advances in the understanding and modeling of the energetic and statistical size effects in the mean statistical sense[1,2]. Computational approaches and simple design code formulas giving better mean predictions have been developed. It now appears, however, that the existing design codes[3,4] and standard practice for concrete and other quasibrittle structures also necessitate major revisions with regard to the effect of structure size and, more generally, degree of brittleness, which depend not only on the size of the structure but also on its structure geometry.

2 PROBABILITY DISTRIBUTIONS OF STRUCTURAL STRENGTH

This study deals with type 1 size effect which occurs for structures failing at macro-fracture initiation from a smooth surface[5]. For this type, material randomness affects both the mean and the scatter of the nominal strength, σ_N. While ductile failure occurs simultaneously along the failure surface and is characterized by Gaussian distribution of structural strength with no size effect, quasibrittle failures propagates, exhibits a strong size effect and, at large sizes, follows extreme value statistics of the weakest-link chain model, which leads to Weibull distribution of structural strength (provided that failure occurs at macro-crack initiation) (Fig. 1a). Based on small- and large-size asymptotic properties recently deduced from the cohesive crack model and nonlocal Weibull theory, the transition of structural strength pdf from small to large sizes is modeled by a chain of fiber bundles, in which each fiber has a Weibull distributed strength and simulates micro-bonds in brittle lower-scale microstructure of a representative volume element (RVE) of the material. To describe structural strength distribution based on the chain-of-bundles model, a composite pdf with a Weibull tail grafted on a Gaussian core is proposed and calibrated (Fig. 1b). For a small structure, the pdf is Gaussian except the far-out left tail, and for the large-size limit totally Weibull. In between, the grafting point moves rightward and the Gaussian core shrinks with increasing structure size.

3 EXTREME VALUES RELIABILITY METHOD (EVRM)

The transition from Gaussian to Weibull pdf causes that the distance from the mean to a point of tolerable failure probability (such as 10^{-7}) nearly doubles (Fig. 1c) as the size of a quasibrittle structure increases. Consequently, the understrength factor, which is prescribed by design codes and cannot be ignored in computer simulation, must be made size dependent, and so must the Cornell and Hasofer-Lind reliability indices for FORM. To relate the reliability index to its value for purely ductile behavior with Gaussian distribution of resistance, the Cornell reliability index, β_C (Eq. 1) may be generalized by introducing the tail offset ratio as follows:

$$\beta_C = \frac{\mu_L - \mu_R}{\sqrt{\theta^2(D)s_R^2 + s_L^2}} \tag{1}$$

where μ_L, μ_R, s_L, s_R are the means and the coefficients of variation of the load and resistance of the structure. Ratio $\theta(D)$ as a function of structure size D can be calculated from the grafted Weibull-Gaussian distributions, the mean size effect law and the chain-of-bundles model as a function of s_R (which is a function of size D), s_L, and the ratio μ_R/μ_L (Fig. 1d).

The Hasofer-Lind reliability index, β_{HL} (Eq. 2) must be revised similarly by modifying the standardized normal variables Y_i':

$$\beta_{HL} = \min\left(\sum_{i=1}^{n} y_i'^2\right)^{1/2}, \qquad Y_i' = \frac{Y_i - \mu_{Y_i}}{\theta_i s_{Y_i}} \tag{2}$$

where Y_i = structure load and resistance parameters; μ_{Y_i}, s_{Y_i} = means and standard deviations; $\theta_i(D) = 1$ for Gaussian variables(load); $\theta_i(D) \geq 1$ for resistance variables.

4 COVERT UNDERSTRENGTH FACTORS

Inseparable from these effects are further problems due to 'covert' understrength factors for material randomness and error of the theory underlying structural analysis (e.g., by finite elements), which are currently implied in brittle failure provisions of concrete design codes, as well as an irrational hidden size effect implied by excessive load factor prescribed for self weight acting alone.

Applications in stochastic finite element analysis based on stratified sampling of random material strength and known type of pdf tail are outlined, and analysis of some past a structural disasters are reviewed.

5 NOVEL RANDOM LATTICE MODEL

A recently developed realistic lattice model for a concrete, having a three-dimensional random microstructure[6,7], is extended for modeling the statistical size effect of concrete by randomizing the material strength and fracture energy. Furthermore, autocorrelation is introduced by assigning the random material strength not directly to the lattice elements, but to the large aggregate pieces in the random microstructure. Each lattice element, which connects two aggregate pieces, is assigned the strength and fracture energy of the aggregate in the largest diameter range (Fig. 1e). Thus, the spatial distributions of the strength and the fracture energy are linked to the microstructure.

This extended lattice model is used to simulate four-point-bending tests of small concrete beams with varying bending span, see Fig. 1f. The results are compared to the original version without random material properties (MP), and to the experiments reported by Koide et al.[8] (Fig. 1g).

The randomized material properties improve the capabilities of the lattice model to simulate the statistical size effect. Nevertheless, a better agreement with the experimental results might be achieved by using for the material properties a random field for which the autocorrelation length is adjusted independently of the size of the aggregates.

6 CONCLUSIONS

- A hybrid pdf with Weibull tail grafted on Gaussian core is used to describe the dependence of pdf of structural strength on structure size, or degree of brittleness.

- A tail offset ratio is introduced to relate the reliability index for quasibrittle structure to its value for ductile structure with Gaussian pdf.

- The lattice model with randomized material properties is capable to simulate the statistical size effect.

REFERENCES

[1] Bažant, Z.P. (2002). *Scaling of Structural Strength.* Hermes Penton Science (Kogan Page Science), London, U.K.

[2] Bažant, Z.P. (2004). "Scaling theory for quasibrittle structural failure." *Proc., National Academy of Sciences*, 101(37), 13397–13399.

[3] Ellingwood, B.R., Galambos, T.V., McGregor, J.G., and Cornell, C.A. (1980). *Development of probability based load criterion for American National Standard A58.* NBS Special Publication 577, U.S. Department of Commerce, Washington, D.C.

[4] Ellingwood, B.R., McGregor, J.G., Galambos, T.V., and Cornell, C.A. (1982). "Probability based load criteria: Load factors and load combinations." *J. of Structural Engrg*, ASCE, 108(ST5), 978–997.

[5] Bažant, Z.P., and Pang, S.D. (2005). " Revision of reliability concepts for quasibrittle structures and size effect on probability distribution of structural strength" *Proc., 9th Int. Conf. on Structural Safety and Reliability* (ICOSSAR), Rome, Italy.

[6] Cusatis, G., Bažant, Z.P., and Cedolin, L. (2003a). "Confinement-shear lattice model for concrete damage in tension and compression: I. Theory." *J. of Engrg. Mech.* ASCE, 2003, 129(12), 1439–1448.

[7] Cusatis, G., Bažant, Z.P., and Cedolin, L. (2003b). "Confinement-shear lattice model for concrete damage in tension and compression: I. Computation and validation." *J. of Engrg. Mech.* ASCE, 2003, 129(12), 1449–1458.

[8] Koide, H., Akita, H., and Tomon, M. (2000). "Probability model of flexural resistance on different lengths of concrete beams", *Appl. of Stat. and Prob.* (Proc., 8th Int. Conf., ICASP-8, held in Sydney, Australia 1999), R. E. Melchers and M.G. Stewart, eds., Balkema, Rotterdam, Vol.2, 1053–1057.

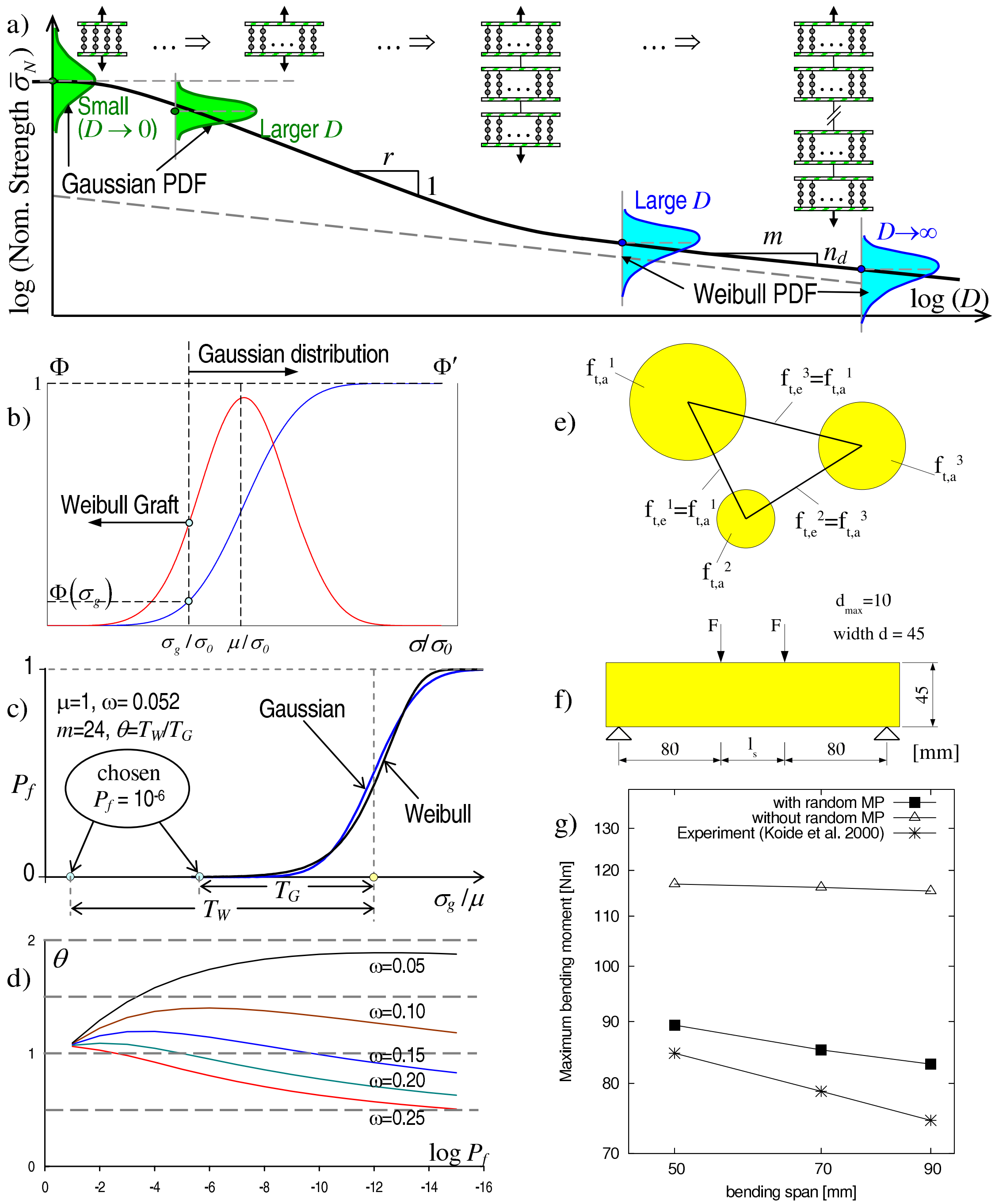

Figure 1: (a) The mean size effect curve for structures failing at macroscopic fracture initiation, and probability distributions of $\bar{\sigma}_N$. (b) pdf and cdf for a Gaussian distribution with Weibull graft at the left tail (plotted accurately for $\mu/\sigma_0 = 0.945$, $s/\sigma_0 = 0.205$, $m = 6$). (c) Comparison of tail offset ratio θ for Weibull and Gaussian distributions. (d) Ratios θ calculated for given failure probability. (e) Schematic illustration of the link between microstructure and random material properties. (f) The geometry and loading setup of the four-point-bending test with bending span lengths l_s of 50, 70 and 90 mm. (g) Comparison of the means of the simulations and of the means of Koide's experimental datas for the three spans used, Series A.