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IDENTIFICATION OF MODELS FOR NONLINEARITY AND HYSTERESIS IN PIEZOELECTRICITY

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Summary. *This paper deals with the identification of material parameters in PDE models for piezoelectricity. In case of large excitations, a nonlinear behaviour has to be taken into account. For this purpose, on one hand, we consider a functional dependence of the elastic stiffness coefficients, the dielectric coefficients and the piezoelectric coupling coefficients on the electric field (and/or the mechanical strain). On the other hand, hysteresis is included into the model by means of a Preisach operator within the governing PDEs. To identify the nonlinear coefficient curves as well as the hysteresis operator, we use inverse methods of iterative type. The proposed approaches in nonlinearity identification can also be applied to different models in mechanics and electromagnetism.*

1 INTRODUCTION

The piezoelectric effect is made use of in a large variety of electromechanical transducers, ranging from ultrasound generation in medical applications to injection valves in cars. The finite element simulation of piezoelectric sensors and actuators is based on a system of partial differential equations

$$\begin{array}{rcll} \rho\frac{\partial^2\vec{d}}{\partial t^2} - \mathcal{B}^T\Big(\mathbf{c}^E\mathcal{B}\vec{d} + \mathbf{e}^T\mathrm{grad}\phi\Big) &=& 0 & \text{in } \Omega \\ -\mathrm{div}\Big(\mathbf{e}\mathcal{B}\vec{d} - \varepsilon^S\mathrm{grad}\phi\Big) &=& 0 & \text{in } \Omega\,. \end{array} \tag{1}$$

where $\mathcal{B}$ is the transposed of the divergence DIV of a dyadic, ρ is the mass density, $\vec{d}$ is the vector of mechanical displacements, and ϕ is the electric potential (cf., e.g., [2], [3]). For this purpose, precise knowledge of the material tensors is essential, namely those of the elastic stiffness coefficients $\mathbf{c}^E \in \mathbb{R}^6_6$, the dielectric coefficients $\varepsilon^S \in \mathbb{R}^3_3$, and the piezoelectric coupling coefficients $\mathbf{e} \in \mathbb{R}^6_3$.

Our task is the identification of these material tensors as coefficients in the piezoelectric system of PDEs, from given measurements of the electric current and/or the mechanical displacement at an electrode attached to the piezoelectric probe.

Here, we especially focus on nonlinearity appearing in the situation of large electric fields and/or mechanical strains, as typical for actuator applications.

2 Nonlinear material parameter curves

In case of large excitations, the material tensors $\mathbf{c}^E$, $\mathbf{e}$, and ε^S will depend on the amplitude of the electric field intensity $\vec{E} = -\mathrm{grad}\phi$ and/or the strain $\vec{S} = \mathcal{B}\vec{d}$. We will here concentrate on electric field dependence and note that strain dependence can be treated analogously.

Since the space dimension can be reduced to one by using appropriately shaped test samples (see [3]), we restrict ourselves to the spatially 1D case of (1),

$$\begin{aligned} \rho d_{,tt} - \Big(c^E(\phi_{,x})d_{,x} + e(\phi_{,x})\phi_{,x}\Big)_{,x} &= 0 \\ -\Big(e(\phi_{,x})d_{,x} - \varepsilon^S\phi_{,x}\Big)_{,x} &= 0\,, \end{aligned} \tag{2}$$

where $x \in (0,L)$, $t \in [0,T]$. Appropriate boundary conditions for an experimental setup with stress free surface, as well as a grounded and an either voltage or charge loaded electrode are

$$\begin{array}{l} (c^E d_{,x} + e\phi_{,x})(0,t) = 0 \\ (c^E d_{,x} + e\phi_{,x})(L,t) = 0 \\ \phi(0,t) = 0 \end{array} \text{ and} \begin{cases} (i)\ (ed_{,x} - \varepsilon^S\phi_{,x})(L,t) = -\frac{q^L(t)}{A} \quad \text{(charge excitation)} \\ \quad \text{or} \\ (ii)\ \phi(L,t) = \phi^L(t) \quad \text{(voltage excitation)}, \end{cases} \tag{3}$$

where A is the surface area covered by the loaded electrode.

As overdetermined data for identifying the curves we use the complementation of the electric Cauchy data at the right hand boundary — obtainable from voltage-current measurements — i.e.,

$$y(t) = \begin{cases} \phi(L,t) & \text{in case (i)} \\ (ed_{,x} - \varepsilon^S\phi_{,x})(L,t) & \text{in case (ii)} \end{cases} \tag{4}$$

Suitably to the experimental characteristics, we perefer to formulate the problem in frequency space rather than in time domain. However, due to the nonlinearity of the coefficients, solutions $\vec{d}$, ϕ of (2) will contain higher harmonics, even when excited at a fixed frequency. To take this into account, we make a multiharmonic ansatz for both field quantities

$$d(x,t) \approx \sum_{k=-N}^{N} e^{\jmath k\omega t}\hat{d}_k(x)\,,\ \phi(x,t) \approx \sum_{k=-N}^{N} e^{\jmath k\omega t}\hat{\phi}_k(x)\,.$$

We insert this ansatz into (2) and integrate with respect to time versus the orthogonal system of time harmonic functions $t \mapsto \frac{\omega}{2\pi}\,e^{-\jmath l\omega t}\chi_{[0,\frac{2\pi}{\omega}]}$. Therewith, we obtain a system of $4N+2$ differential equations, for the space dependent functions $\hat{d}_k$, $\hat{\phi}_k$, $k = -N, \ldots N$. In general, this system still contains time integration, which we wish to avoid. This can be done by making a polynomial ansatz for each of the searched for curves c^E, e, ε^S

$$c^E(E) = \sum_{p=0}^{P_c} a_p^c E^p, \quad e(E) = \sum_{p=0}^{P_e} a_p^e E^p, \quad \varepsilon^S(E) = \sum_{p=0}^{P_\epsilon} a_p^\epsilon E^p.$$

By means of the multinomial theorem, we can factor out the time harmonic exponential terms and make use of orthogonality to obtain the nonlinear system of differential equations

$$\left.\begin{aligned} -\rho\omega^2 l^2 \hat{d}_l - \sum_{k=-N}^{N}\left(\sum_{p=0}^{P_c} a_p^c \bar{c}_{l-k}^p \hat{d}_{k,x} + \sum_{p=0}^{P_e} a_p^e \bar{c}_{l-k}^p \hat{\phi}_{k,x}\right)_{,x} &= 0 \\ -\sum_{k=-N}^{N}\left(\sum_{p=0}^{P_e} a_p^e \bar{c}_{l-k}^p \hat{d}_{k,x} - \sum_{p=0}^{P_e} a_p^\epsilon \bar{c}_{l-k}^p \hat{\phi}_{k,x}\right)_{,x} &= 0 \end{aligned}\right\} \text{ for } l = -N, \dots N$$

for the space dependent functions $\hat{d}_k$, $\hat{\phi}_k$, $k = -N, \dots N$, with boundary conditions corresponding to (3). Here, the coefficients $\bar{c}_\Delta^p$ are given by

$$\bar{c}_\Delta^p = \sum_{\mathbf{p}\in\mathcal{I}(p,\Delta)} \binom{p}{\mathbf{p}} \prod_{m=-N}^{N} \hat{\phi}_{m,x}^{p_m}$$

with $\binom{p}{\mathbf{p}} = \frac{p!}{p_{-N}!\cdots p_N!}$, and $\mathcal{I}(p,\Delta) = \{\mathbf{p} \in \mathbb{N}_0^{2N+1} \,|\sum_{k=-N}^{N} p_k = p \wedge \sum_{k=-N}^{N} k p_k = \Delta\}$.

The given measurements can be incorporated by matching boundary values of solutions of (2) to the multiharmonic coefficients $\hat{y}_{-N} \dots \hat{y}_N$ of $y = y^{meas}$ in $y(t) \approx \sum_{k=-N}^{N} e^{\jmath k\omega t}\hat{y}_k$ with y according to (4).

Our problem is now reduced to identification of the coefficients $a_0^c, \dots, a_{P_c}^c, a_0^e, \dots, a_{P_e}^e$, $a_0^\epsilon, \dots, a_{P_\epsilon}^\epsilon$ in (2) from these given measurements. We define the forward operator G, that maps the vector of coefficients $\underline{a} = (a_0^c, \dots, a_{P_c}^c, a_0^e, \dots, a_{P_e}^e, a_0^\epsilon, \dots, a_{P_\epsilon}^\epsilon)$ to the vector of computed measurements $\hat{y}_{-N} \dots \hat{y}_N$ according to (4). Therewith, we can write the identification problem as a — possibly overdetermined — nonlinear system of equations

$$G(\underline{a}) = \underline{\hat{y}}\,,$$

that we solve by a Gauss-Newton iteration, see [4] for details.

3 Hysteresis

An additional nonlinear phenomenon that is typical for the large-signal behaviour of piezoelectric transducers is hysteresis, (see, e.g., [1], and Fig. 1, left). For simplicity of exposition, we here consider preliminary a static model of electric polarization, neglecting the mechanical coupling:

$$\mathrm{div}\vec{D} = 0\,, \quad \vec{D} = \varepsilon_0\vec{E} + \vec{P}\,,$$

where $\vec{D}$ denotes the electric flux density, $\vec{E} = -\mathrm{grad}\phi$ the electric field and $\vec{P}$ the polarization vector that depends on the electric field in a hysteretic way. Further simplifying the situation to the one-dimensional case, we arrive at the boundary value problem

$$-\left(\varepsilon_0\phi_{,x} - \mathcal{P}[-\phi_x]\right)_{,x} = 0\,, \quad \phi(0,t) = 0\,, \quad \phi(L,t) = \phi^L(t)\,. \tag{5}$$

Rate independent memory effects can be modeled in a very general and flexible way by a Preisach operator that is determined by a bivariate weight function w in

$$p(t) = \mathcal{P}[e](t) = \iint_{-1\leq\beta\leq\alpha\leq 1} w(\alpha,\beta)\mathcal{R}_{\alpha,\beta}[e](t)\, d\alpha d\beta \tag{6}$$

where $\{\mathcal{R}_{\alpha,\beta}\}_{-1\leq\beta\leq\alpha\leq 1}$ is a family of elementary switching operators, see Fig. 1, (right). This can be applied in (5) by inserting $\frac{E(x,\cdot)}{E_{sat}}$, $\frac{P(x,\cdot)}{P_{sat}}$ in place of the normed quantities $|p| \leq 1$, $|e| \leq 1$, respectively, for each space point x. For identifying the Preisach operator (more precisely a discrete version w_h of the weight function) from additional boundary measurements $y(t) = P(L,t)$ we propose two iterative schemes. The first one is based on alternating iterations for solving the Dirichlet boundary value problem with data ϕ^L for ϕ, and the Neumann boundary value problem with data y for w_h. The second one is based on a Newton type approach for simultaneously resolving both the Dirichlet and the Neumann problem for ϕ and w_h.

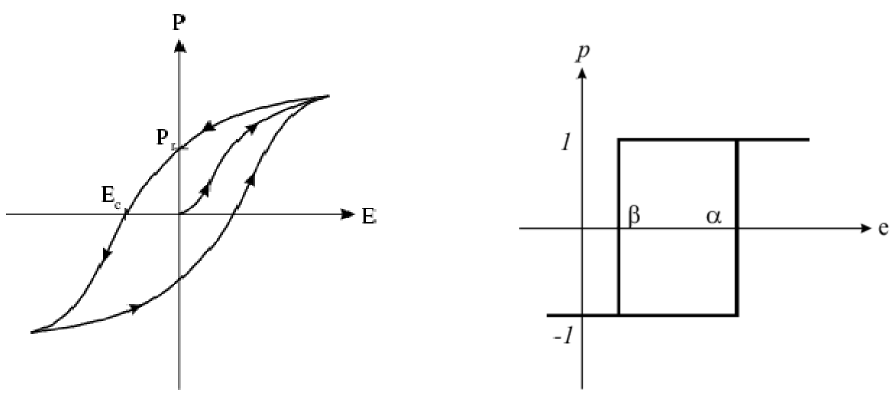

Figure 1: Hysteresis curve of polarization (left) composed of elementary switching operators $\mathcal{R}_{\alpha\beta}$ (right) in the Preisach model (6).

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