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FIELD TRANSFER IN NONLINEAR STRUCTURAL MECHANICS BASED ON DIFFUSE APPROXIMATION

D. Brancherie*, **P. Villon***, **A. Ibrahimbegovic**†,
A. Rassineux* **and P. Breitkopf***

*Laboratoire Roberval, Université de Technologie de Compiègne
BP 20529 60205 Compiègne Cedex, France
e-mail: delphine.brancherie@utc.fr, web page: http://www.utc.fr/lrm

†LMT-Cachan, Ecole Normale Supérieure de Cachan
61 avenue du Président Wilson 94235 Cachan Cedex
e-mail: ai@lmt.ens-cachan.fr, Web page: http://www.lmt.ens-cachan.fr

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Summary. *In this work, we present an adaptive remeshing technique for non linear structures. We will focus, in this paper, essentially on the phase of the strategy consisting in the reconstruction of fields from an old discretisation to a new one. The proposed operator is based on diffuse approximation and has the particularity to guarantee local equilibrium preservation, stress admissibility and energy conservation. Some numerical results obtained for damage structures are presented.*

1 Introduction

For a large class of problems such as forming process, strain localization or crack propagation, the remeshing of the computational domain is mandatory in order to obtain an optimal discretization with respect to the description of different solution fields.

The efficiency of such adaptive techniques requires reliable transfer operators to continue the calculation on the new mesh with a minimum discrepancy. Some key points of the quality of such a transfer operator are, among others, consistency with constitutive equations [1], conservation of equilibrium, or for some applications, energy conservation.

The developments proposed here consist in an adaptative technique dealing with nonlinear materials (e.g. damage materials). The transfer operator is based on diffuse approximation and preserves local equilibrium, stress admissibility and ensures conservation of dissipated and strain energy. Hence, the reconstruction of fields relies on diffuse approximation [3], [4] and results in the resolution of optimisation problems under constraints.

2 Damage constitutive model

Applications considered here concern the description of rupture for damage structures. For such structures, macro-cracks responsible for rupture are initiated from micro-cracks which can be represented by a local continuum damage model [2].

We present here an isotropic associated model used to reproduce the nucleation and development of micro-cracks in the bulk material. The internal variables are the forth order compliance tensor denoted $\overline{\mathbf{D}}$ and the hardening variable $\bar{\xi}$ (defining damage state). The model is construted in the same way as associated plasticity by invoking the principles of thermodynamics and the principle of maximum dissipation. The damage function considered here is $\bar{\phi}(\boldsymbol{\sigma}, \bar{q}) = \sqrt{\boldsymbol{\sigma} : \mathbf{D}^e : \boldsymbol{\sigma}} - \frac{1}{\sqrt{E}}(\sigma_f - \bar{q}) \leq 0$ where $\bar{q}$ is the $\bar{\xi}$ dual variable. In those conditions, the instantaneous disspation can be written as : $0 \leq \mathcal{D} = \bar{\boldsymbol{\varepsilon}} : \overline{\mathbf{D}}^{-1} : \bar{\boldsymbol{\varepsilon}} + \bar{q}\dot{\bar{\xi}}$.

3 Field transfer strategy

For such a model, the continuation of the calculation needs the reconstruction of the damage state, the stress state and the displacement field. The transfer operator proposed is then decomposed into three steps. First step consists in the "topology" and state transfer. At that stage, damage variables are rebuilt on the new mesh in order to define damaged, damaging and sane areas imposing in the same time conservation of dissipated energy. Second step consists in the reconstruction of the stress field imposing local equilibrium verification and stress admissibility. Finally, third step consists in the reconstruction of the displacement field imposing boundary conditions and kinematic compatibility.

3.1 Damage state transfer

Reconstruction of the damage state $\bar{\xi}$ and its evolution is the first step of the operator proposed. By appealing to diffuse approximation, the scalar variable transfered is decomposed on a polynomial basis of approximation (here, of degree one) $\tilde{\mathbf{v}} = \mathbf{p}^{\mathbf{T}}\mathbf{a} = \begin{bmatrix} 1 & x & y \end{bmatrix} \mathbf{a}$

The field on the new mesh is constructed as the solution of a minimisation problem:

$$\min_{\mathbf{a}} J(\mathbf{a}) = \sum_{i \in V(x)} w(x_i, x) \|\tilde{\mathbf{v}}(x_i) - \mathbf{v}(x_i)\|^2 \tag{1}$$

where $V(x)$ is a neighborhood of the point x and x_i are the points of the old mesh where $\mathbf{v}$ is known. The weight function $w(x_i, x)$ ensures the continuity of the reconstructed fields. The latter are used to define the damaged ($\bar{\xi} \neq 0$) and damaging ($\Delta\bar{\xi} \neq 0$) areas. The renormalisation of the field on the damaged areas allows to preserve dissipated energy.

3.2 Stress reconstruction

Two different strategies have been adopted depending on whether the point of approximation is in a damaging area or not.

3.2.1 Damaging area

Damaging areas are defined by $\Delta\bar{\xi} \neq 0$ which imposes, by Kuhn Tuker conditions, stress admissibility $\bar{\phi}(\boldsymbol{\sigma}, \bar{q}) = 0$. Reconstruction of the stress field is then carried out by imposing local equilibrium conservation ($\mathbf{div}\, \boldsymbol{\sigma} = 0$ and stress admissibility).

Then, the stress field is approximated by a field of the form:

$$\boldsymbol{\sigma}^{\text{new}} = \begin{bmatrix} 1 & 0 & 0 & x & 0 & y & 0 \\ 0 & 1 & 0 & 0 & x & 0 & y \\ 0 & 0 & 1 & -y & 0 & 0 & -x \end{bmatrix} \begin{bmatrix} \theta_1 \\ ... \\ \theta_7 \end{bmatrix} = \mathbf{P}^T(\mathbf{x})\boldsymbol{\theta} \qquad (2)$$

ensuring local equilibrium in a diffuse sense. $\boldsymbol{\sigma}^{new}$ is obtained by computing $\boldsymbol{\theta}$ as the solution of a minimisation problem under the constraint $\bar{\phi}(\boldsymbol{\sigma}, \bar{q}) = 0$ rewritten as a quadratic constraint:

$$\min_{\boldsymbol{\theta}^T \mathbf{C} \boldsymbol{\theta} = 1} J_{\mathbf{x}}(\boldsymbol{\theta}) \quad \text{with} \quad J_{\mathbf{x}}(\boldsymbol{\theta}) = \frac{1}{2} \sum_{i \in V(\mathbf{x})} w(\mathbf{x}_i - \mathbf{x}) \| \mathbf{P}^T(\mathbf{x}_i - \mathbf{x})\boldsymbol{\theta} - \boldsymbol{\sigma}^{old}(\mathbf{x}_i)^2 \| \qquad (3)$$

3.2.2 Non damaging areas

Non damaging areas are characterized by $\Delta\bar{\xi} = 0$ and $\bar{\phi}(\boldsymbol{\sigma}, \bar{q}) \leq 0$. In this zones, a first step is to rebuild, by diffuse approximation, a stress field verifying the local equilibrium equation. After renormalization of the stress field in order to preserve strain energy, a cellular automata checking and restablishing stress admissibility is used. The latter restablishes stress admissibility by a sequence of elastic loading/unloading on a patch of elements at constant strain energy.

3.3 Displacement field reconstruction

In order to continue the computation, we also need to rebuild the displacement field on the new discretization. The field transferred on the new mesh may be compatible with the strain field $\boldsymbol{\varepsilon}^{new}$ obtained from the state equation and the rebuilt compliance and stress fields. It also has to verify the essential boundary conditions. The displacement field on the new mesh is then built as the solution of a minimisation problem under boundary conditions constraint:

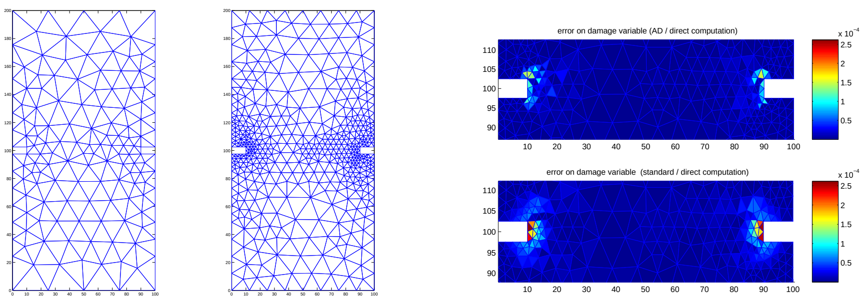
$$\min_{\mathbf{u}|_\Gamma = \bar{\mathbf{u}}} \frac{1}{2} \left(w_1 \| \mathbf{u}^{\text{new}} - p_{AD}(\mathbf{u}^{\text{old}}) \|^2_{L^2(\Omega)} + w_2 h^2 \| \boldsymbol{\nabla}^{\text{s}} \mathbf{u}^{\text{new}} - \boldsymbol{\varepsilon}^{\text{new}} \|^2_{L^2(\Omega)} \right) \qquad (4)$$

where $p_{AD}(\mathbf{u}^{\text{old}})$ is the diffuse approximation projection of the displacement from the old mesh to the new one. w_1 and w_2 are weights defining the importance with which the displacement has to verify kinematic compatibility or proximity to projected displacement. Finally, h is the length of elements and $\bar{\mathbf{u}}$ is the imposed displacement on the essential boundary Γ.

4 Numerical results

In this section, we present some results obtained by performing the trasnfer described previously. We consider a notched beam under traction. Two remeshings are performed during the computation in order to adapt the discretization to the solution (figure 1).

We present the results obtained in terms of damage variable map for the first remeshing comparing results obtained by standard transfer or proposed transfer to the solution obtained by direct computation (figure 1-(c)).



(a) First mesh

(b) New mesh

(c) Equilibrium recovery - proposed/standard transfer

Figure 1: Initial and first adapted mesh - Error compared to direct computation after field transfer

We can note that the results given by the proposed transfer operator give better evaluation of the damage state than the standard transfer. The damage zone is well captured and numerical diffusion is limited.

5 Conclusion

The work presented here deals with the development of a transfer operator for non linear materials. The operator proposed is based on diffuse approximation and preserves local equilibrium, stress admissibility and energetic quantities definig the non-linear state of the structure (strain and dissipated energy).

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