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A THREE-DIMENSIONAL CONTACT ELEMENT BASED ON THE MOVING FRICTION CONE APPROACH AND THE ELLIPTICAL COULOMB LAW

Lovre Krstulović-Opara* **and Peter Wriggers†**

*Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture
University of Split
R. Boškovića bb., HR-21000 Split, Croatia
E-mail: Lovre.Krstulovic-Opara@fesb.hr – Web page: http//:www.fesb.hr/kk

† Institut für Baumechanik und Numerische Mechanik
Universität Hannover
Appelstrasse 9A, D-30167 Hannover, Germany
E-mail: wriggers@ibnm.uni-hannover.de - Web page: http//:www.ibnm.uni-hannover.de

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Summary. *The paper presents elliptical Coulomb law where the friction surface is defined with two principal friction coefficients and corresponding direction, what enables description of surfaces showing biaxial frictional response. The Moving Friction Cone formulation is based on the contact constraint described using a single gap vector that enables significantly simpler, shorter and faster element code.*

1 INTRODUCTION

Materials showing different friction intensity regarding sliding direction, e.g. contact with composite surfaces or contact with surfaces showing unidirectional roughness can be modeled using an elliptical frictional surface law. The Coulomb cone, characterized by a single friction coefficient can be modified by creating an elliptical surface defined by two principal friction coefficients, $\mu_1 = tg\,\alpha_1$ and $\mu_2 = tg\,\alpha_2$ (Fig. 1) and the corresponding angle φ defining the orientation. The Moving Friction Elliptical Cone (MFEC) formulation is derived from the Moving Friction Cone (MFC) formulation [1,2]. Generally, when describing a contact between two 3D bodies, the master-node to slave-surface approach based on the contact constraint in the form of normal and tangential gap is often used. Contrary to this, in the MFC formulation the contact constraint is defined using a single gap vector (Fig. 2). In the case of stick, the spring-back effect of the elastic-like behavior is characterizing the case when the slave node is still within the Coulomb frictional cone. For the slip case the cone is moving to the position where the slave node on the surface is evaluated from the fact that the gap vector is perpendicular to the normal on the Coulomb cone surface. Thus, the spring-back effect is again pushing the penetrated slave node back to the master surface.

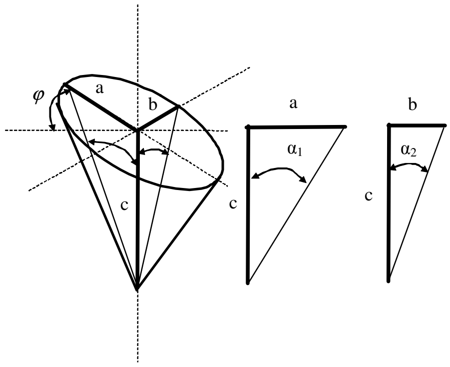

Figure 1: The elliptical Coulomb cone

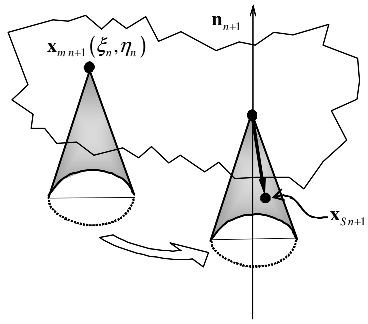

Figure 2: The definition of gap vector

2 THE CONTACT GEOMETRY DESCRIPTION

Based on a tetrahedral continuum element, a flat triangular contact surface (Fig. 3) is defined within the current configuration $\mathbf{x}_{n+1} = \mathbf{X} + \mathbf{u}_{n+1}$ in the parametric form as:

$$\mathbf{x}_{n+1}(\xi_{n+1},\eta_{n+1}) = \begin{bmatrix} \xi_{n+1} & \eta_{n+1} & (1-\xi_{n+1}-\eta_{n+1}) \end{bmatrix} \cdot \begin{bmatrix} x_{11\,n+1} & x_{21\,n+1} & x_{31\,n+1} \\ x_{12\,n+1} & x_{22\,n+1} & x_{32\,n+1} \\ x_{13\,n+1} & x_{23\,n+1} & x_{33\,n+1} \end{bmatrix}. \tag{1}$$

Supposing the known solution point for the last converged stage $\mathbf{x}_{m\,n}(\xi_n,\eta_n)$ (defined by parameters ξ_n,η_n for time $t=t_n$), the solution point is mapped to the current configuration

$$\mathbf{x}_{m\,n+1}(\xi_n,\eta_n) = \begin{bmatrix} \xi_n & \eta_n & (1-\xi_n-\eta_n) \end{bmatrix} \cdot \begin{bmatrix} x_{11\,n+1} & x_{21\,n+1} & x_{31\,n+1} \\ x_{12\,n+1} & x_{22\,n+1} & x_{32\,n+1} \\ x_{13\,n+1} & x_{23\,n+1} & x_{33\,n+1} \end{bmatrix}. \tag{2}$$

The contact condition is formulated as:

$$\mathbf{g}_{S\,n+1}(\xi_n,\eta_n) \cdot \mathbf{n}_{n+1} \le 0, \quad \mathbf{n}_{n+1} = (\mathbf{x}_{1\,n+1} - \mathbf{x}_{3\,n+1}) \times (\mathbf{x}_{2\,n+1} - \mathbf{x}_{3\,n+1}), \tag{3}$$

where the elastic gap vector $\mathbf{g}_{S\,n+1}(\xi_n,\eta_n)$ is defined between the slave node $\mathbf{x}_{S\,n+1}$ and the last converged stage in the current configuration, i.e. $\mathbf{g}_{S\,n+1}(\xi_n,\eta_n) = \mathbf{x}_{S\,n+1} - \mathbf{x}_{m\,n+1}(\xi_n,\eta_n)$. In the case of contact the stick case is supposed to occur and the trial traction vector is defined as:

$$\mathbf{t}^{tr}_{n+1}(\xi_n,\eta_n) = \varepsilon\, \mathbf{g}_{S\,n+1}(\xi_n,\eta_n), \tag{4}$$

where ε is a constant penalty parameter. Projection of the trial traction vector in the normal and tangential direction defines the normal pressure and the trial traction vector

$$p_{N\,n+1}(\xi_n,\eta_n) = \mathbf{t}^{tr}_{n+1}(\xi_n,\eta_n) \cdot \frac{\mathbf{n}_{n+1}}{\sqrt{\mathbf{n}_{n+1}\cdot\mathbf{n}_{n+1}}}, \quad \mathbf{t}^{tr}_{T\,n+1}(\xi_n,\eta_n) = \mathbf{t}^{tr}_{n+1}(\xi_n,\eta_n) - p_{N\,n+1}(\xi_n,\eta_n) \cdot \frac{\mathbf{n}_{n+1}}{\sqrt{\mathbf{n}_{n+1}\cdot\mathbf{n}_{n+1}}}. \tag{5}$$

When the components of trial traction vector are defined, the supposed (trial) stick state can be verified by checking the elliptical Coulomb criterion

$$f_S^{tr} = \left\| \mathbf{t}_{T\,n+1}^{tr}\left(\xi_n,\eta_n\right) \right\| - \sqrt{\frac{\mu_1^{\,2}\,\mu_2^{\,2}}{\mu_2^{\,2}\,\cos^2\varphi + \mu_1^{\,2}\,\sin^2\varphi}}\, Abs\left[p_{N\,n+1}\left(\xi_n,\eta_n\right) \right] \le 0\,. \tag{6}$$

where φ is the angle between the principal axis vector $\mathbf{p}_\mu$ (defines the direction of principal friction coefficient μ_1) and the sliding direction vector

$$\mathbf{t}_{S\,n+1} = \mathbf{x}_{S\,n+1} - \mathbf{e}_{N\,n+1}\left[\left(\mathbf{x}_{S\,n+1} - \mathbf{x}_{3\,n+1}\right)\cdot \mathbf{e}_{N\,n+1}\right] - \mathbf{x}_{m\,n+1}\left(\xi_n,\eta_n\right). \tag{7}$$

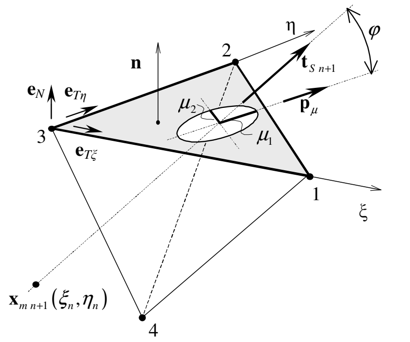

Figure 3: The principal axis direction

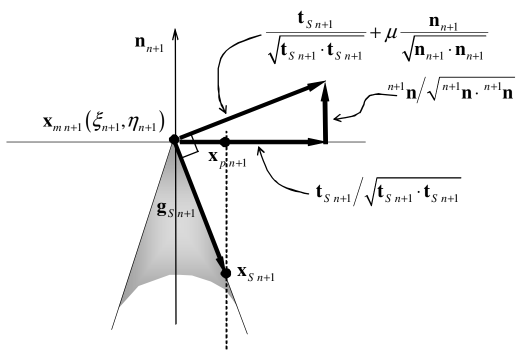

Figure 4: The perpendicularity condition

If relation (6) is not satisfied, sliding occurs. The new position of the cone, i.e. of the cone's tip $\mathbf{x}_{m\,n+1}\left(\xi_{n+1},\eta_{n+1}\right)$, is evaluated from the perpendicularity condition (Fig. 4)

$$\mathbf{g}_{S\,n+1}\left(\xi_{n+1},\eta_{n+1}\right)\cdot\left(\frac{\mathbf{t}_{S\,n+1}}{\sqrt{\mathbf{t}_{S\,n+1}\cdot\mathbf{t}_{S\,n+1}}} + \mu\frac{\mathbf{n}_{n+1}}{\sqrt{\mathbf{n}_{n+1}\cdot\mathbf{n}_{n+1}}}\right) = 0 \tag{8}$$

and the additional condition

$$\left[\mathbf{g}_{S\,n+1}\left(\xi_{n+1},\eta_{n+1}\right)\times\mathbf{t}_{S\,n+1}\right]\cdot\mathbf{n}_{n+1} = 0\,, \tag{9}$$

where $\mathbf{g}_{S\,n+1}\left(\xi_{n+1},\eta_{n+1}\right) = \mathbf{x}_{S\,n+1} - \mathbf{x}_{m\,n+1}\left(\xi_{n+1},\eta_{n+1}\right)$ is the elastic gap vector. To solve this relation a Newton-Raphson iterative procedure is performed within the each iteration.

3 THE DEFINITION OF THE RESIDUAL VECTOR AND THE TANGENT MATRIX

In the case where no sliding occurred, i.e. stick, the residual vector and the tangent matrix are obtained explicitly. The residual vector and the tangent matrix for the stick case are:

$$\Psi_{stick\ i}^{c} = \varepsilon\,\mathbf{g}_{S\,n+1}\frac{\partial\,\mathbf{g}_{S\,n+1}}{\partial\,u_{i\,n+1}}\,,\quad K_{stick\ ij}^{c} = \frac{\partial\,\Psi_{stick\ i}^{c}}{\partial\,u_j}\,,\quad i = 1,\ldots 12,\ \ j = 1,\ldots 12\,. \tag{10}$$

For the case when sliding occurred, the new solution parameters ξ_{n+1},η_{n+1} are evaluated using the Newton-Raphson procedure. The residual vector is defined as:

$$\Psi^{c}_{slip\ i} = \varepsilon\ \mathbf{g}_{S\ n+1}(\xi_{n+1},\eta_{n+1}) \cdot \left(\frac{\partial\ \mathbf{x}_{S\ n+1}}{\partial\ u_{i\ n+1}} - \frac{\partial\ \mathbf{x}_{m\ n+1}(\xi_{n+1},\eta_{n+1})}{\partial\ u_{i\ n+1}} \right), \qquad i = 1,\ldots 12\ . \tag{11}$$

$$K^{c}_{slip ij} = \frac{\partial\ \Psi^{c}_{slip\ i}}{\partial\ u_j} + \frac{\partial\ \Psi^{c}_{slip\ i}}{\partial \xi_{n+1}} \frac{\partial \xi_{n+1}}{\partial\ u_j} + \frac{\partial\ \Psi^{c}_{slip\ i}}{\partial \eta_{n+1}} \frac{\partial \eta_{n+1}}{\partial u_j} \quad , \qquad i = 1,\ldots 12, \quad j = 1,\ldots 12 \quad ,$$

When linearising the tangent matrix (11), the Newton-Raphson procedure of relations (8) and (9) has to be taken into account such that

$$\frac{\partial\ \xi_{n+1}}{\partial\ u_j} = -\frac{\dfrac{\partial F_{NR}}{\partial\ u_j}}{\dfrac{\partial F_{NR}}{\partial\ \xi_{n+1}}}\ , \qquad \frac{\partial\ \eta_{n+1}}{\partial\ u_j} = -\frac{\dfrac{\partial F_{NR}}{\partial\ u_j}}{\dfrac{\partial F_{NR}}{\partial\ \eta_{n+1}}}\ , \qquad F_{NR} = \begin{bmatrix} \mathbf{g}_{S\ n+1}(\xi_{n+1},\eta_{n+1}) \cdot \left(\dfrac{\mathbf{t}_{S\ n+1}}{\sqrt{\mathbf{t}_{S\ n+1} \cdot \mathbf{t}_{S\ n+1}}} + \mu \dfrac{\mathbf{n}_{n+1}}{\sqrt{\mathbf{n}_{n+1} \cdot \mathbf{n}_{n+1}}} \right) \\ \left[\mathbf{g}_{S\ n+1}(\xi_{n+1},\eta_{n+1}) \times \mathbf{t}_{S\ n+1} \right] \cdot \mathbf{n}_{n+1} \end{bmatrix} . \tag{12}$$

4 NUMERICAL EXAMPLE

Using the same sliding cube example[1,2], the MFEC (μ_1=0.1, μ_2=0.08) results for φ=0°, φ=45°, φ=90° are compared with the MFC formulation when μ=0.1 and μ=0.08. The total tangential reaction for the nodes where the cube is clamped and moved is depicted in Fig. 6.

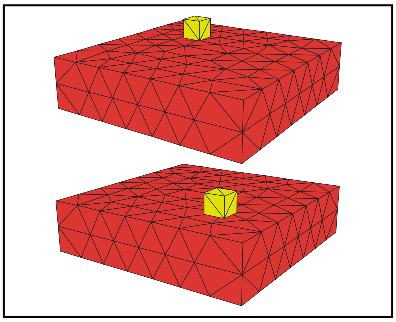

Figure 5: The sliding cube

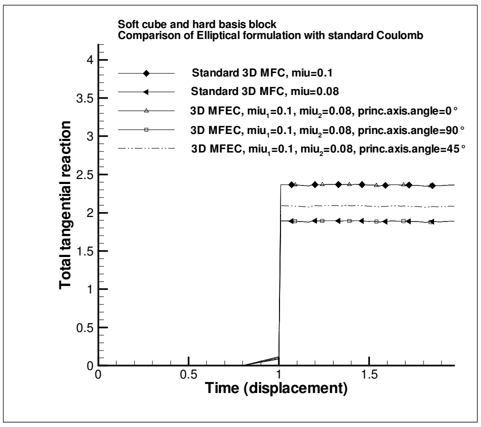

Figure 6: The total tangential reaction

5 CONCLUSIONS

The significant simplification of this single gap vector in the MFEC/MFC approach is reducing the code size and the complexity of a contact element what results into the faster contact element routine and the better overall performance of the simulation. The use of a single penalty parameter reduces the effort of finding proper penalty parameter such that penetration is minimal and ill-conditioning is avoided.

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